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Vapor Flows Along a Plane Condensed Phase with Weak Condensation in the Presence of a Noncondensable Gas

Satoshi Taguchi,[1] Kazuo Aoki,[2] and Vladimir Latocha[3]

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A steady flow of a vapor in a half space condensing at incidence onto a plane condensed phase is considered in the case where another gas that does not condense (the noncondensable gas) is present near the condensed phase. A systematic asymptotic analysis of the Boltzmann equation for hard-sphere molecules is performed in the case where condensation is weak, and the relation among the parameters of the vapor flow at infinity, those associated with the plane condensed phase, and the amount of the noncondensable gas is derived in an analytical form. The result supplements the numerical result for the relation for arbitrarily strong condensation obtained on the basis of a model Boltzmann equation and under the restriction that the vapor molecules are mechanically identical with the noncondensable-gas molecules [Taguchi *et al.*, Phys. Fluids **15**: 689 (2003)].

KEY WORDS: Boltzmann equation, kinetic theory of gases, binary gas mixture, Knudsen layer, evaporation and condensation
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1. INTRODUCTION

Let us consider a steady flow of a vapor around its condensed phase of arbitrary shape, on the surface of which evaporation or condensation may take place. The continuum limit, where the Knudsen number goes to zero, of such a system is investigated in ref. 1, where the appropriate fluid-dynamic system (the compressible

[1] Fachbereich Mathematik und Informatik, Johannes Gutenberg-Universität, 55099 Mainz, Germany
[2] Department of Mechanical Engineering and Science and Advanced Research Institute of Fluid Science and Engineering, Kyoto University, Kyoto 606-8501, Japan; e-mail: aoki@aero.mbox.media.kyoto-u.ac.jp
[3] Institut de Mathématiques Elie Cartan, Université Henri Poincaré-Nancy I, B.P. 239, 54506 Vandoeuvre-lés-Nancy, France

Euler equations and their boundary conditions) are derived by means of a systematic asymptotic analysis of the Boltzmann equation and its boundary condition (see also ref. 2). Recently, the analysis of ref. 1 was extended to the case where a small amount of another gas that neither evaporates nor condenses on the condensed phase (noncondensable gas) is contained in the system[(3)], and it was shown that a trace of the noncondensable gas causes a significant effect on the vapor flow, through the boundary condition for the Euler equations on the condensing surface.

The boundary condition for the Euler equations on the condensing surface is derived by solving the half-space problem of condensation of the nonlinear Boltzmann equation in the presence of the noncondensable gas.[(3)] More specifically, the boundary condition is equivalent to the relationship among the parameters of the vapor at infinity, those associated with the plane condensed phase, and the amount of the noncondensable gas confined near the condensed phase. This half-space problem has been investigated in refs. 4–7, and the relationship has been established numerically.

In these references, the relationship is obtained under the condition that the mechanical property of the vapor molecules is the same as that of the noncondensable-gas molecules because it leads to various advantages, such as the simplification of analysis and the drastic reduction of the amount of computation.[(4)] However, this at the same time reduces the applicability of the numerical boundary condition to the practical problems. In addition, the numerical boundary condition is obtained on the basis of the model Boltzmann equation for a gas mixture proposed by Garzó, Santos, and Brey (GSB model).[(8)]

Now, let us give a look at the half-space problem of condensation of a vapor in the absence of the noncondensable gas. This problem has comprehensively been investigated both by numerical[(9−14)] and analytical[(15−19)] methods. In refs. 15 and 16, the relationship among the parameters of the vapor at infinity and those associated with the condensed phase is derived analytically under the condition of weak condensation.

In view of this fact, it is worth trying a similar analysis of the half-space problem of condensation in the presence of the noncondensable gas when the condensation is weak. In the present paper, we perform such an analysis of the Boltzmann equation to derive the relationship among the parameters in a more explicit form in a more general case under the restriction that the condensation is weak.

2. FORMULATION OF THE PROBLEM

2.1. Problem

Consider a vapor in a half space $X_1 > 0$ bounded by a stationary plane condensed phase of the vapor located at $X_1 = 0$, where X_i is a rectangular coordinate

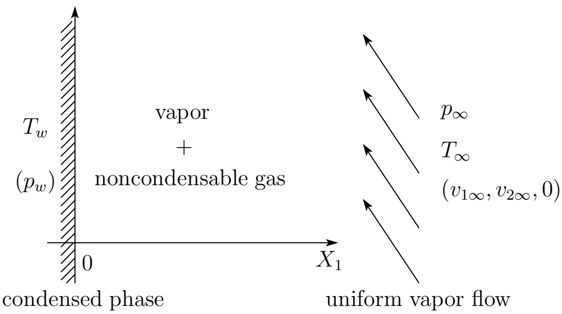

Fig. 1. Vapor flow condensing on a plane condensed phase in the presence of a noncondensable gas.

system. There is a uniform vapor flow at infinity toward the condensed phase with velocity $v_{i\infty} = (v_{1\infty},\ v_{2\infty},\ 0)$ ($v_{1\infty} < 0$, $v_{2\infty} \geq 0$), temperature T_∞, and pressure p_∞ (or molecular number density $n_\infty = p_\infty/kT_\infty$, where k is the Boltzmann constant). The condensed phase is kept at a constant and uniform temperature T_w. Steady condensation of the vapor is taking place on the condensed phase, and another gas neither condensing nor evaporating on the condensed phase, which we call the noncondensable gas, is confined near the condensed phase by the condensing vapor flow. (See Fig. 1) We investigate the steady behavior of the vapor and the noncondensable gas on the basis of kinetic theory, under the following assumptions: (i) the molecules of the vapor and those of the noncondensable gas are elastic hard spheres; (ii) the vapor molecules leaving the condensed phase are distributed according to the Maxwellian distribution describing the saturated equilibrium state at rest at temperature T_w; (iii) the noncondensable-gas molecules leaving the condensed phase are distributed according to the Maxwellian distribution with temperature T_w and flow velocity 0, and there is no net particle flow across the condensed phase (diffuse reflection); (iv) the condensation is slow. The assumption (ii), which is a conventional condition for evaporation and condensation, has been justified to some extent by recent studies based on molecular dynamics[(20)] and an Enskog–Vlasov kinetic system.[(21)]

For later convenience, let us denote by p_w the saturated pressure of the vapor at temperature T_w and by n_w the corresponding molecular number density ($n_w = p_w/kT_w$). In the following, we assign the label A to the vapor (it will also be called A-component) and B to the noncondensable gas (it will also be called B-component).

2.2. Basic Equation

We first introduce the basic notations: ξ_i is the molecular velocity, F^α the velocity distribution function of the α-component ($\alpha = A$ corresponds to the vapor and $\alpha = B$ to the noncondensable gas); n^α is the molecular number density, ρ^α the mass density, T^α the temperature, p^α the pressure, and $v_i^\alpha = (v_1^\alpha,\ v_2^\alpha,\ 0)$ the flow velocity of the α-component; n is the molecular number density, ρ the mass density, T the temperature, p the pressure, and $v_i = (v_1,\ v_2,\ 0)$ the flow velocity of the total mixture; m^α and d^α are the mass and diameter of a molecule of the α-component.

Then, the Boltzmann equation for a binary mixture in the present steady and spatially one-dimensional problem reads[22–24]

$$\xi_1 \frac{\partial F^\alpha}{\partial X_1} = \sum_{\beta=A,B} J^{\beta\alpha}(F^\beta,\ F^\alpha), \qquad (\alpha = A, B), \tag{1}$$

where $J^{\beta\alpha}(F^\beta,\ F^\alpha)$ is the collision term that expresses the effect of molecular collisions between molecules of the α and β-components on the change of F^α. Its explicit form will be given in the dimensionless form in the next subsection.

The boundary condition on the condensed phase is given as follows:

$$F^A = \frac{n_w}{(2\pi k T_w/m^A)^{3/2}} \exp\left(-\frac{\xi_j^2}{2kT_w/m^A}\right), \tag{2a}$$

$$F^B = \frac{\sigma_w^B}{(2\pi k T_w/m^B)^{3/2}} \exp\left(-\frac{\xi_j^2}{2kT_w/m^B}\right), \tag{2b}$$

for $\xi_1 > 0$ at $X_1 = 0$, where

$$\sigma_w^B = -\left(\frac{2\pi m^B}{kT_w}\right)^{1/2} \int_{\xi_1<0} \xi_1 F^B(X_1 = 0, \xi_i) d^3\xi, \tag{3}$$

with $d^3\xi = d\xi_1 d\xi_2 d\xi_3$. The condition at infinity is

$$F^A \to \frac{n_\infty}{(2\pi k T_\infty/m^A)^{3/2}} \exp\left(-\frac{(\xi_1 - v_{1\infty})^2 + (\xi_2 - v_{2\infty})^2 + \xi_3^2}{2kT_\infty/m^A}\right), \tag{4a}$$

$$F^B \to 0, \tag{4b}$$

as $X_1 \to \infty$.

2.3. Dimensionless Variables

We first choose the following mean free path l_w and speed c_w as the reference length and speed:

$$l_w = \frac{1}{\sqrt{2}\pi(d^A)^2 n_w}, \qquad c_w = \left(\frac{2kT_w}{m^A}\right)^{1/2}. \tag{5}$$

Here, l_w and c_w are, respectively, the mean free path and the most probable speed of the vapor molecules in the equilibrium state at rest with molecular number density n_w and temperature T_w. Then, we introduce the following dimensionless parameters

$$\hat{v}_{i\infty} = v_{i\infty}/c_w, \quad \hat{n}_\infty = n_\infty/n_w, \quad \hat{T}_\infty = T_\infty/T_w, \quad \hat{p}_\infty = p_\infty/p_w, \tag{6a}$$

$$\hat{m}^\alpha = m^\alpha/m^A, \qquad \hat{d}^\alpha = d^\alpha/d^A, \qquad (\alpha = A,\ B), \tag{6b}$$

and dimensionless variables

$$x_i = (2/\sqrt{\pi})(X_i/l_w), \quad \zeta_i = \xi_i/c_w, \quad \hat{F}^\alpha = (c_w^3/n_w)F^\alpha, \tag{7a}$$

$$\hat{n}^\alpha = n^\alpha/n_w, \quad \hat{\rho}^\alpha = \rho^\alpha/m^A n_w, \quad \hat{T}^\alpha = T^\alpha/T_w, \tag{7b}$$

$$\hat{p}^\alpha = p^\alpha/p_w, \quad \hat{v}_i^\alpha = v_i^\alpha/c_w, \quad (\hat{v}_3^\alpha = 0), \tag{7c}$$

$$\hat{n} = n/n_w, \quad \hat{\rho} = \rho/m^A n_w, \quad \hat{T} = T/T_w, \tag{7d}$$

$$\hat{p} = p/p_w, \quad \hat{v}_i = v_i/c_w, \quad (\hat{v}_3 = 0). \tag{7e}$$

The Boltzmann equation (1) is then written in the following dimensionless form:

$$\zeta_1 \frac{\partial \hat{F}^\alpha}{\partial x_1} = \sum_{\beta=A,B} \hat{K}^{\beta\alpha} \hat{J}^{\beta\alpha}(\hat{F}^\beta, \hat{F}^\alpha), \quad (\alpha = A,\ B), \tag{8}$$

where

$$\hat{J}^{\beta\alpha}(f, g) = \frac{1}{4\sqrt{2\pi}} \int [f(\zeta_{*i}^{\beta\alpha})g(\zeta_i^{\beta\alpha}) - f(\zeta_{*i})g(\zeta_i)]|e_j \hat{V}_j| d\Omega(e_i) d^3\zeta_*, \tag{9a}$$

$$\zeta_i^{\beta\alpha} = \zeta_i + \frac{\hat{\mu}^{\beta\alpha}}{\hat{m}^\alpha}(e_j \hat{V}_j)e_i, \quad \zeta_{*i}^{\beta\alpha} = \zeta_{*i} - \frac{\hat{\mu}^{\beta\alpha}}{\hat{m}^\beta}(e_j \hat{V}_j)e_i, \tag{9b}$$

$$\hat{\mu}^{\beta\alpha} = 2\hat{m}^\alpha \hat{m}^\beta/(\hat{m}^\alpha + \hat{m}^\beta), \qquad \hat{V}_i = \zeta_{*i} - \zeta_i, \tag{9c}$$

$$\hat{K}^{\beta\alpha} = [(\hat{d}^\alpha + \hat{d}^\beta)/2]^2, \qquad d^3\zeta_* = d\zeta_{*1} d\zeta_{*2} d\zeta_{*3}. \tag{9d}$$

Here, e_i is a unit vector, ζ_{*i} the variable of integration corresponding to ζ_i, and $d\Omega(e_i)$ the solid angle element in the direction of e_i. The domain of integration in Eq. (9a) is the whole space of ζ_{*i} and all directions of e_i. The collision integral $\hat{J}^{\beta\alpha}(f, g)$ has the properties summarized in Appendix A.

The dimensionless form of the boundary conditions (2a) and (2b) on the condensed phase is given as

$$\hat{F}^A = \pi^{-3/2} \exp(-\zeta_j^2), \tag{10a}$$

$$\hat{F}^B = (\hat{m}^B/\pi)^{3/2} \hat{\sigma}_w^B \exp(-\hat{m}^B \zeta_j^2), \tag{10b}$$

for $\zeta_1 > 0$ at $x_1 = 0$, where

$$\hat{\sigma}_w^B = -2(\pi \hat{m}^B)^{1/2} \int_{\zeta_1 < 0} \zeta_1 \hat{F}^B(x_1 = 0,\ \zeta_i) d^3\zeta, \tag{11}$$

with $d^3\zeta = d\zeta_1 d\zeta_2 d\zeta_3$, and that of the conditions (4a) and (4b) at infinity is

$$\hat{F}^A \to \hat{n}_\infty (\pi \hat{T}_\infty)^{-3/2} \exp\left(-\frac{(\zeta_1 - \hat{v}_{1\infty})^2 + (\zeta_2 - \hat{v}_{2\infty})^2 + \zeta_3^2}{\hat{T}_\infty} \right), \tag{12a}$$

$$\hat{F}^B \to 0, \tag{12b}$$

as $x_1 \to \infty$.

The macroscopic quantities $\hat{n}^\alpha$, $\hat{\rho}^\alpha$, $\hat{v}_i^\alpha$, $\hat{p}^\alpha$, $\hat{T}^\alpha$, $\hat{n}$, $\hat{\rho}$, $\hat{v}_i$, $\hat{p}$, and $\hat{T}$ are defined as follows:

$$\hat{n}^\alpha = \int \hat{F}^\alpha d^3\zeta, \qquad \hat{\rho}^\alpha = \hat{m}^\alpha \hat{n}^\alpha, \tag{13a}$$

$$\hat{v}_i^\alpha = \frac{1}{\hat{n}^\alpha} \int \zeta_i \hat{F}^\alpha d^3\zeta, \quad (\hat{v}_3^\alpha = 0), \tag{13b}$$

$$\hat{p}^\alpha = \hat{n}^\alpha \hat{T}^\alpha = \frac{2}{3} \hat{m}^\alpha \int (\zeta_j - \hat{v}_j^\alpha)^2 \hat{F}^\alpha d^3\zeta, \tag{13c}$$

$$\hat{n} = \int \sum_{\beta = A,B} \hat{F}^\beta d^3\zeta, \quad \hat{\rho} = \int \sum_{\beta = A,B} \hat{m}^\beta \hat{F}^\beta d^3\zeta, \tag{13d}$$

$$\hat{v}_i = \frac{1}{\hat{\rho}} \int \zeta_i \sum_{\beta = A,B} \hat{m}^\beta \hat{F}^\beta d^3\zeta, \quad (\hat{v}_3 = 0), \tag{13e}$$

$$\hat{p} = \hat{n} \hat{T} = \frac{2}{3} \int (\zeta_j - \hat{v}_j)^2 \sum_{\beta = A,B} \hat{m}^\beta \hat{F}^\beta d^3\zeta. \tag{13f}$$

The domain of integration of the integrals with respect to ζ_i in Eqs. (13a)–(13f) is the whole space of ζ_i. The same rule applies to all the integrals with respect to ζ_i in this paper unless the contrary is stated. The macroscopic quantities for the total mixture are expressed in terms of those for individual components as

$$\hat{n} = \sum_{\beta=A,B} \hat{n}^\beta, \quad \hat{\rho} = \sum_{\beta=A,B} \hat{\rho}^\beta, \quad \hat{\rho}\hat{v}_i = \sum_{\beta=A,B} \hat{\rho}^\beta \hat{v}_i^\beta, \tag{14a}$$

$$\hat{p} = \sum_{\beta=A,B} \left[\hat{p}^\beta + \frac{2}{3}\hat{\rho}^\beta\left(\hat{v}_j^\beta - \hat{v}_j\right)^2\right]. \tag{14b}$$

It should be noted that in the literature, the pressure $\hat{p}^\alpha$ and temperature $\hat{T}^\alpha$ of each component are often defined in a different way, i.e., by Eq. (13c) with $\hat{v}_j^\alpha$ replaced by $\hat{v}_j$ of Eq. (13e). Then, the pressure $\hat{p}$ of the total mixture becomes the simple sum of $\hat{p}^A$ and $\hat{p}^B$ rather than Eq. (14b).

The boundary-value problem, Eqs. (8), (10a), (10b), (12a), and (12b), contains the dimensionless parameters $\hat{T}_\infty$, $\hat{p}_\infty$ (or $\hat{n}_\infty$), $\hat{v}_{1\infty}$, $\hat{v}_{2\infty}$, $\hat{m}^B$, and $\hat{d}^B$. In addition to these, we need a parameter that specifies the amount of the noncondensable gas in the system. In our previous works,[4–7] the following Γ is used:

$$\Gamma = \frac{2}{\sqrt{\pi}} \frac{1}{n_\infty l_\infty} \int_0^\infty n^B dX_1, \tag{15}$$

where l_∞ is the mean free path of the vapor molecules in the equilibrium state at rest with temperature T_∞ and molecular number density n_∞, which is, for hard-sphere molecules, given by

$$l_\infty = \frac{1}{\sqrt{2}\pi (d^A)^2 n_\infty}. \tag{16}$$

For hard-sphere molecules, Γ becomes

$$\Gamma = \int_0^\infty \hat{n}^B dx_1, \tag{17}$$

because $n_w l_w = n_\infty l_\infty$ holds.

The integration of Eq. (8) over the whole ζ_i space leads to $\hat{n}^\alpha \hat{v}_1^\alpha = \text{const}$ because the right-hand side vanishes in the integration (Appendix A). For the noncondensable gas, $\hat{n}^B \hat{v}_1^B = 0$ holds on the condensed phase because of the diffuse reflection (10b) and (11) or at infinity because of Eq. (12b). Therefore, $\hat{n}^B \hat{v}_1^B = 0$ or

$$\hat{v}_1^B = 0, \tag{18}$$

holds identically for $x_1 \geq 0$.

2.4. Parameter Relation

The boundary-value problem defined by Eqs. (8), (10a), (10b), (12a), and (12b) is characterized by the dimensionless parameters $\hat{p}_\infty$, $\hat{T}_\infty$, $\hat{v}_{1\infty}$, $\hat{v}_{2\infty}$, and

Γ in addition to the parameters $\hat{m}^B$ and $\hat{d}^B$ of the molecular property. These parameters are not independent but should satisfy certain relations in order that the problem has a (time-independent) solution. These relations have been clarified in ref. 4 and subsequently in refs. 5–7 under the assumption that the molecules of the vapor and those of the noncondensable gas are mechanically identical ($\hat{m}^B = \hat{d}^B = 1$ in the case of hard-sphere molecules). Let us introduce

$$M_{n\infty} = \frac{|v_{1\infty}|}{(5kT_\infty/3m^A)^{1/2}} = \left(\frac{6}{5\hat{T}_\infty}\right)^{1/2} |\hat{v}_{1\infty}|, \tag{19a}$$

$$M_{t\infty} = \frac{v_{2\infty}}{(5kT_\infty/3m^A)^{1/2}} = \left(\frac{6}{5\hat{T}_\infty}\right)^{1/2} \hat{v}_{2\infty}, \tag{19b}$$

where $M_{n\infty}$ and $M_{t\infty}$ are, respectively, the Mach numbers based on the normal and tangential components of the flow velocity at infinity. By the use of these Mach numbers (instead of $\hat{v}_{1\infty}$ and $\hat{v}_{2\infty}$) and of the original ratios p_∞/p_w and T_∞/T_w (instead of $\hat{p}_\infty$ and $\hat{T}_\infty$), the relations are summarized as

$$\frac{p_\infty}{p_w} = \mathcal{F}_s\left(M_{n\infty}, M_{t\infty}, \frac{T_\infty}{T_w}, \Gamma\right), \quad (M_{n\infty} < 1), \tag{20a}$$

$$\frac{p_\infty}{p_w} \geq \mathcal{F}_b\left(M_{n\infty}, M_{t\infty}, \frac{T_\infty}{T_w}, \Gamma\right), \quad (M_{n\infty} = 1), \tag{20b}$$

$$\frac{p_\infty}{p_w} > \mathcal{F}_b\left(M_{n\infty}, M_{t\infty}, \frac{T_\infty}{T_w}, \Gamma\right), \quad (M_{n\infty} > 1). \tag{20c}$$

The functions $\mathcal{F}_s$ and $\mathcal{F}_b$ have been obtained numerically using the GSB model of the Boltzmann equation in refs. 4–7.

The aim of the present study is to derive analytical or explicit expressions of $\mathcal{F}_s$ for hard-sphere molecules without the restriction that the molecules of the two species are mechanically identical, under the assumption of weak condensation, i.e., for $M_{n\infty} \ll 1$ or $|\hat{v}_{1\infty}| \ll 1$.

3. SLOWLY VARYING SOLUTION AND FLUID-DYNAMIC EQUATIONS

In the present half-space problem, there is no *a priori* length scale of variation other than the mean free path l_w (or l_∞). However, it was shown in the case of a pure vapor that, when condensation is weak, there appears another length scale of variation, which is much longer than l_w and may be characterized by $l_w/|\hat{v}_{1\infty}|$.[2,15,16] The solution with this length scale of variation, which may be called the slowly-varying solution and is described in a fluid-dynamic way (see below), plays an essential role in the present analysis. The slowly-varying solution also appears in the case of transonic condensation.[17] It should be mentioned that the notion of the slowly-varying solution was applied recently to the analysis of half-space evaporation/condensation problems for a mixture of two vapors.[25,26]

3.1. Integral Equations

Let us put $\epsilon = |\hat{v}_{1\infty}| \ll 1$. We first assume that the length scale of variation of the overall flow field is l_w/ϵ, which is much longer than the mean free path l_w, and look for such a solution $\hat{F}_H^\alpha$ (slowly-varying solution), where the subscript H is attached to denote the slowly-varying solution. For this purpose, we introduce the space coordinate y contracted by ϵ, i.e.,

$$y = \epsilon x_1, \tag{21}$$

and assume that $\hat{F}_H^\alpha = \hat{F}_H^\alpha(y, \zeta_i)$ [or $\partial \hat{F}_H^\alpha/\partial y = O(\hat{F}_H^\alpha)$]. Then the Boltzmann equation (8) is recast as

$$\zeta_1 \frac{\partial \hat{F}_H^\alpha}{\partial y} = \frac{1}{\epsilon} \sum_{\beta=A,B} \hat{K}^{\beta\alpha} \hat{J}^{\beta\alpha}(\hat{F}_H^\beta, \hat{F}_H^\alpha), \qquad (\alpha = A, B), \tag{22}$$

We try to obtain the slowly-varying solution in the form of a simple power-series expansion in ϵ, i.e., a Hilbert-type expansion:

$$\hat{F}_H^\alpha = \hat{F}_{H(0)}^\alpha + \hat{F}_{H(1)}^\alpha \epsilon + \hat{F}_{H(2)}^\alpha \epsilon^2 + \cdots . \tag{23}$$

Correspondingly, any macroscopic quantity h_H of the slowly-varying solution [i.e., h represents $\hat{n}^\alpha$, $\hat{v}_i^\alpha$ ($\hat{v}_3^\alpha = 0$), $\hat{T}^\alpha$, $\hat{n}$, $\hat{v}_i$ ($\hat{v}_3 = 0$), $\hat{T}$, etc.] is expanded as

$$h_H = h_{H(0)} + h_{H(1)}\epsilon + h_{H(2)}\epsilon^2 + \cdots . \tag{24}$$

The coefficients $h_{H(m)}$ in Eq. (24) are expressed in terms of $\hat{F}_{H(n)}^\alpha$ $(n \leq m)$ by substituting Eqs. (23) and (24) in Eqs. (13a)–(13f) with $h = h_H$ and $\hat{F}^\alpha = \hat{F}_H^\alpha$ (note that the relation between h and $\hat{F}^\alpha$ is generally nonlinear) and equating the coefficients of the same power of ϵ (the result is summarized in Appendix B).

If we substitute Eq. (23) into Eq. (22) and arrange the powers of ϵ, we obtain the following sequence of integral equations for the coefficients $\hat{F}_{H(m)}^\alpha$, which can be solved from the lowest order:

$$\sum_{\beta=A,B} \hat{K}^{\beta\alpha} \hat{J}^{\beta\alpha}\left(\hat{F}^{\beta}_{H(0)},\ \hat{F}^{\alpha}_{H(0)}\right) = 0, \tag{25}$$

$$\sum_{\beta=A,B} \hat{K}^{\beta\alpha} \left[\hat{J}^{\beta\alpha}\left(\hat{F}^{\beta}_{H(m)},\ \hat{F}^{\alpha}_{H(0)}\right) + \hat{J}^{\beta\alpha}\left(\hat{F}^{\beta}_{H(0)},\ \hat{F}^{\alpha}_{H(m)}\right)\right]$$
$$= \zeta_1 \frac{\partial \hat{F}^{\alpha}_{H(m-1)}}{\partial y} - \sum_{\beta=A,B} \hat{K}^{\beta\alpha} \sum_{n=1}^{m-1} \hat{J}^{\beta\alpha}\left(\hat{F}^{\beta}_{H(m-n)},\ \hat{F}^{\alpha}_{H(n)}\right), \tag{26}$$

where $m = 1,\ 2,\ \ldots$, and $\sum_1^0 = 0$ when $m = 1$ in Eq. (26).

Equation (25) shows that $\hat{F}^{\alpha}_{H(0)}$ are local Maxwellians with common flow velocity and temperature, which are expressed in terms of $\hat{n}^{\alpha}_{H(0)}$, $\hat{v}_{iH(0)}$ ($\hat{v}_{3H(0)} = 0$), and $\hat{T}_{H(0)}$ as

$$\hat{F}^{\alpha}_{H(0)} = \hat{n}^{\alpha}_{H(0)} \left(\frac{\hat{m}^{\alpha}}{\pi \hat{T}_{H(0)}}\right)^{3/2} \exp\left(-\frac{\hat{m}^{\alpha}(\zeta_j - \hat{v}_{jH(0)})^2}{\hat{T}_{H(0)}}\right). \tag{27}$$

However, in view of the condition at infinity (12a), where $\hat{v}_{1\infty} = -\epsilon$, we should assume that

$$\hat{v}_{1H(0)} = \hat{v}^{\alpha}_{1H(0)} = 0. \tag{28}$$

We further assume that

$$\hat{n}^{B}_{H(0)} = 0, \qquad (\text{or } \hat{F}^{B}_{H(0)} = 0), \tag{29}$$

for the following reason. We are considering the case where the parameter Γ is of the order of unity. However, Γ is expressed as $\Gamma = (1/\epsilon)\int(\hat{n}^{B}_{H(0)} + \hat{n}^{B}_{H(1)}\epsilon + \cdots)dy$ because of Eqs. (17), (21), and (24). If the condition (29) is not assumed, Γ becomes of the order of $1/\epsilon$. With these assumptions, the analysis can be carried out consistently. To summarize, $\hat{F}^{A}_{H(0)}$ and $\hat{F}^{B}_{H(0)}$ become

$$\hat{F}^{A}_{H(0)} = \frac{\hat{n}_{H(0)}}{(\pi \hat{T}_{H(0)})^{3/2}} \exp\left(-\frac{\zeta_1^2 + (\zeta_2 - \hat{v}_{2H(0)})^2 + \zeta_3^2}{\hat{T}_{H(0)}}\right), \tag{30}$$

$$\hat{F}^{B}_{H(0)} = 0. \tag{31}$$

Consequently, Eq. (26) with $\alpha = A$ and that with $\alpha = B$ are recast respectively as

$$\hat{J}^{AA}\big(\hat{F}^A_{H(m)}, \hat{F}^A_{H(0)}\big) + \hat{J}^{AA}\big(\hat{F}^A_{H(0)}, \hat{F}^A_{H(m)}\big)$$
$$= \zeta_1 \frac{\partial \hat{F}^A_{H(m-1)}}{\partial y} - \sum_{\beta=A,B} \hat{K}^{\beta A} \sum_{n=1}^{m-1} \hat{J}^{\beta A}(\hat{F}^\beta_{H(m-n)}, \hat{F}^A_{H(n)})$$
$$- \hat{K}^{BA} \hat{J}^{BA}(\hat{F}^B_{H(m)}, \hat{F}^A_{H(0)}), \qquad (32)$$

$$\hat{K}^{AB} \hat{J}^{AB}(\hat{F}^A_{H(0)}, \hat{F}^B_{H(m)})$$
$$= \zeta_1 \frac{\partial \hat{F}^B_{H(m-1)}}{\partial y} - \sum_{\beta=A,B} \hat{K}^{\beta B} \sum_{n=1}^{m-1} \hat{J}^{\beta B}(\hat{F}^\beta_{H(m-n)}, \hat{F}^B_{H(n)}). \qquad (33)$$

Equation (33) with $m = 1$ reduces to

$$\hat{J}^{AB}\big(\hat{F}^A_{H(0)}, \hat{F}^B_{H(1)}\big) = 0. \qquad (34)$$

It is easy to show that the solution of Eq. (34) is given by a local Maxwellian with the same flow velocity and temperature as those of $\hat{F}^A_{H(0)}$, i.e.,

$$\hat{F}^B_{H(1)} = \hat{n}^B_{H(1)} \left(\frac{\hat{m}^B}{\pi \hat{T}_{H(0)}} \right)^{3/2} \exp\left(-\hat{m}^B \frac{\zeta_1^2 + (\zeta_2 - \hat{v}_{2H(0)})^2 + \zeta_3^2}{\hat{T}_{H(0)}} \right). \qquad (35)$$

It should be noted that, when $\hat{F}^B_{H(0)} = 0$, the $\hat{v}^B_{2H(0)}$ and $\hat{T}^B_{H(0)}$ are given by (Eqs. B2b) and (B2c) (with $\alpha = B$), multiplied by $\hat{n}^B_{H0}$, and are expressed as the moments of $\hat{F}^B_{H(1)}$:

$$\hat{v}^B_{2H(0)} = \frac{1}{\hat{n}^B_{H(1)}} \int \zeta_2 \hat{F}^B_{H(1)} d^3\zeta, \qquad (36a)$$

$$\hat{T}^B_{H(0)} = \frac{2\hat{m}^B}{3\hat{n}^B_{H(1)}} \int \big[\zeta_1^2 + \big(\zeta_2 - \hat{v}^B_{2H(0)}\big)^2 + \zeta_3^2\big] \hat{F}^B_{H(1)} d^3\zeta. \qquad (36b)$$

From Eqs. (B1b) and (B1c) ($\alpha = A$) with Eq. (30) and Eqs. (36a) and (36b) with Eq. (35), we have

$$\hat{v}^A_{2H(0)} = \hat{v}^B_{2H(0)} = \hat{v}_{2H(0)}, \qquad \hat{T}^A_{H(0)} = \hat{T}^B_{H(0)} = \hat{T}_{H(0)}. \qquad (37)$$

Then, $\hat{F}^A_{H(m)}$ ($m = 1, 2, ...$) are determined by Eq. (32), and $\hat{F}^B_{H(m)}$ ($m = 2, 3, ...$) by Eq. (33), if they are solved alternately. Since $\hat{F}^A_{H(0)}$ is a local Maxwellian, the left-hand side of Eq. (32) is essentially the linearized collision operator (for a single-component gas) on $\hat{F}^A_{H(m)}$, whereas that of Eq. (33) is the linear collision operator on $\hat{F}^B_{H(m)}$.

3.2. Fluid-Dynamic Equations

Since the homogeneous equation corresponding to Eq. (32), i.e.,

$$\hat{J}^{AA}\big(f, \hat{F}^A_{H(0)}\big) + \hat{J}^{AA}\big(\hat{F}^A_{H(0)}, f\big) = 0, \tag{38}$$

has the nontrivial solutions $f = (\hat{F}^A_{H(0)},\ \zeta_i \hat{F}^A_{H(0)},\ \zeta_j^2 \hat{F}^A_{H(0)})$, the right-hand side of Eq. (32) should satisfy the following solvability conditions:

$$\int \big(1,\ \zeta_i,\ \zeta_j^2\big)[\text{R.H.S of Eq. (32)}]d^3\zeta = 0, \tag{39}$$

which can be transformed to the following form ($m = 1,\ 2,\ \ldots$):

$$\mathcal{S}^A_{0(m-1)}: \quad \frac{d}{dy}\int \zeta_1 \hat{F}^A_{H(m-1)} d^3\zeta = 0, \tag{40a}$$

$$\mathcal{S}^A_{i(m-1)}: \quad \frac{d}{dy}\sum_{\beta=A,B} \hat{m}^\beta \int \zeta_1 \zeta_i \hat{F}^\beta_{H(m-1)} d^3\zeta = 0, \tag{40b}$$

$$\mathcal{S}^A_{4(m-1)}: \quad \frac{d}{dy}\sum_{\beta=A,B} \hat{m}^\beta \int \zeta_1 \zeta_j^2 \hat{F}^\beta_{H(m-1)} d^3\zeta = 0. \tag{40c}$$

Equation (40a) follows directly from the first condition of Eq. (39) because of the property (A1). Equations (40b) and (40c) follow respectively from the second and third conditions of Eq. (39) if Eq. (33) as well as the properties (A2) and (A3) is taken into account. Then, the solution $\hat{F}^A_{H(m)}$ to Eq. (32) is expressed in the following form:

$$\hat{F}^A_{H(m)} = \hat{F}^A_{H(0)}\big(c^A_{0(m)} + c^A_{j(m)}\zeta_j + c^A_{4(m)}\zeta_j^2\big) + \Psi^A_{(m)}, \tag{41}$$

where $c^A_{0(m)}$, $c^A_{i(m)}$, and $c^A_{4(m)}$ are unknown functions of y, and $\Psi^A_{(m)}$ is the particular solution satisfying the condition

$$\int (1,\ \zeta_i,\ \zeta_j^2)\Psi^A_{(m)} d^3\zeta = 0. \tag{42}$$

With Eq. (42), $c^A_{0(m)}$, $c^A_{i(m)}$, and $c^A_{4(m)}$ are uniquely expressed in terms of $\hat{n}^A_{H(n)}$, $\hat{v}^A_{iH(n)}$, and $\hat{T}^A_{H(n)}$ ($n \leq m$).

On the other hand, the homogeneous equation corresponding to Eq. (33), i.e.,

$$\hat{J}^{AB}(\hat{F}^A_{H(0)}, f) = 0, \tag{43}$$

has the nontrivial solution $f = \hat{F}^B_{H(1)}$ [see Eq. (34)]. Therefore, the right-hand side of Eq. (33) should satisfy the following solvability condition:

$$\int [\text{R.H.S of Eq. (33)}] d^3\zeta = 0, \tag{44}$$

which reduces to

$$\mathcal{S}^B_{0(m-1)}: \quad \frac{d}{dy} \int \zeta_1 \hat{F}^B_{H(m-1)} d^3\zeta = 0, \tag{45}$$

because of the property (A1). Then, the solution of Eq. (33) is obtained in the form

$$\hat{F}^B_{H(m)} = c^B_{0(m)} \hat{F}^B_{H(1)} + \Psi^B_{(m)}, \tag{46}$$

where $c^B_{0(m)}$ is an unknown function of y, and $\Psi^B_{(m)}$ is the particular solution satisfying the condition

$$\int \Psi^B_{(m)} d^3\zeta = 0. \tag{47}$$

With this condition, $c^B_{0(m)}$ is expressed as $c^B_{0(m)} = \hat{n}^B_{H(m)}/\hat{n}^B_{H(1)}$.

By using Eqs. (30), (31), (35), (41), and (46) successively in Eqs. (40) and (45), we obtain sets of ordinary differential equations for macroscopic variables, as we will see. These equations are the so-called fluid-dynamic-type equations. Here we only give the explicit forms of $\hat{F}^A_{H(1)}$ and $\hat{F}^B_{H(2)}$ and the fluid-dynamic equations obtained in this stage.

If we substitute Eq. (30) into the solvability condition (40) with $m = 1$ (recall that $\hat{F}^B_{H(0)} = 0$), $\mathcal{S}^A_{1(0)}$ gives

$$\frac{d\hat{p}_{H(0)}}{dy} = 0, \tag{48}$$

where $\hat{p}_{H(0)} = \hat{p}^A_{H(0)}$, and the other conditions are satisfied automatically.

With the condition (48), Eq. (32) with $m = 1$ is solved to give

$$\begin{aligned}\hat{F}^A_{H(1)} = \hat{F}^A_{H(0)} \Bigg[& \frac{\hat{p}^A_{H(1)}}{\hat{p}_{H(0)}} + 2\frac{\hat{v}^A_{jH(1)}}{\hat{T}^{1/2}_{H(0)}}\tilde{\zeta}_j + \left(\tilde{\zeta}^2 - \frac{5}{2}\right)\frac{\hat{T}^A_{H(1)}}{\hat{T}_{H(0)}} \\ & - \frac{1}{\hat{p}_{H(0)}}\frac{d\hat{T}_{H(0)}}{dy}\tilde{\zeta}_1 A(\tilde{\zeta}) - \frac{\hat{T}^{1/2}_{H(0)}}{\hat{p}_{H(0)}}\frac{d\hat{v}_{2H(0)}}{dy}\tilde{\zeta}_1\tilde{\zeta}_2 B(\tilde{\zeta})\Bigg],\end{aligned} \tag{49}$$

where

$$\tilde{\zeta}_i = \frac{\zeta_i - \hat{v}_{iH(0)}}{\hat{T}^{1/2}_{H(0)}}, \quad \tilde{\zeta} = (\tilde{\zeta}_j^2)^{1/2}, \quad \hat{v}_{1H(0)} = \hat{v}_{3H(0)} = 0, \quad \hat{v}^A_{3H(1)} = 0, \tag{50}$$

and the functions $A(\zeta)$ and $B(\zeta)$ in Eq. (49) are defined in Appendix C. It follows from Eqs. (B2f), (B2g) [with Eqs. (B2d) and (B2h)], (29), and (37) that

$$\hat{v}^A_{iH(1)} = \hat{v}_{iH(1)}, \qquad \hat{T}^A_{H(1)} = \hat{T}_{H(1)}. \tag{51}$$

The substitution of the explicit forms of $\hat{F}^A_{H(1)}$ [Eq. (49) with Eq. (51)] and $\hat{F}^B_{H(1)}$ [Eq. (35)] into Eqs. (40a)–(40c) with $m = 2$ leads to the following fluid-dynamic equations:

$$\frac{d}{dy}\left(\hat{n}_{H(0)}\hat{v}_{1H(1)}\right) = 0, \tag{52a}$$

$$\frac{d}{dy}\sum_{\alpha=A,B}\hat{p}^\alpha_{H(1)} = 0, \tag{52b}$$

$$\frac{d}{dy}\left(\hat{n}_{H(0)}\hat{v}_{1H(1)}\hat{v}_{2H(0)} - \frac{\gamma_1}{2}\hat{T}^{1/2}_{H(0)}\frac{d\hat{v}_{2H(0)}}{dy}\right) = 0, \tag{52c}$$

$$\frac{d}{dy}\left[\hat{n}_{H(0)}\hat{v}_{1H(1)}\left(\frac{5}{2}\hat{T}_{H(0)} + \hat{v}^2_{2H(0)}\right) - \frac{5}{4}\gamma_2\hat{T}^{1/2}_{H(0)}\frac{d\hat{T}_{H(0)}}{dy} - \gamma_1\hat{T}^{1/2}_{H(0)}\hat{v}_{2H(0)}\frac{d\hat{v}_{2H(0)}}{dy}\right] = 0, \tag{52d}$$

where γ_1 and γ_2 are the constants defined in Appendix C and have the numerical values

$$\gamma_1 = 1.270042, \qquad \gamma_2 = 1.922284. \tag{53}$$

The γ_1 and γ_2 are, respectively, related to the viscosity and thermal conductivity of the vapor (see Appendix C). Here, Eqs. (52a), (52b), (52c), and (52d) follow from $\mathcal{S}^A_{0(1)}$, $\mathcal{S}^A_{1(1)}$, $\mathcal{S}^A_{2(1)}$, and $\mathcal{S}^A_{4(1)}$, respectively.

On the other hand, $\mathcal{S}^B_{0(1)}$ [Eq. (45)] with Eq. (35) is automatically satisfied. Therefore, Eq. (33) with $m = 2$ can be solved to give the following $\hat{F}^B_{H(2)}$:

$$\begin{aligned}\hat{F}^B_{H(2)} = \hat{F}^B_{H(1)}\Bigg[&\frac{\hat{n}^B_{H(2)}}{\hat{n}^B_{H(1)}} + 2\hat{m}^B\frac{\hat{v}_{jH(1)}}{\hat{T}^{1/2}_{H(0)}}\tilde{\zeta}_j + \left(\hat{m}^B\tilde{\zeta}^2 - \frac{3}{2}\right)\frac{\hat{T}_{H(1)}}{\hat{T}_{H(0)}}\\ &- \frac{1}{\hat{p}_{H(0)}}\frac{d\hat{T}_{H(0)}}{dy}\tilde{\zeta}_1 A^B(\tilde{\zeta}; X^A = 1) - \frac{\hat{T}^{1/2}_{H(0)}}{\hat{p}_{H(0)}}\frac{d\hat{v}_{2H(0)}}{dy}\tilde{\zeta}_1\tilde{\zeta}_2 B^B(\tilde{\zeta}; X^A = 1)\\ &+ \frac{1}{\hat{p}_{H(0)}}\frac{1}{\hat{\rho}^B_{H(1)}}\frac{d\hat{p}^B_{H(1)}}{dy}\tilde{\zeta}_1 D^{(A)B}(\tilde{\zeta}; X^A = 1)\Bigg],\end{aligned} \tag{54}$$

where $A^B(\zeta; X^A)$, $B^B(\zeta; X^A)$, and $D^{A(B)}(\zeta; X^A)$ are the functions of ζ depending on a parameter X^A ($0 \le X^A \le 1$) as well as $\hat{m}^B$ $(= m^B/m^A)$ and $\hat{d}^B$ $(= d^B/d^A)$, which are defined in ref. 27 and computed in ref. 28 [they are denoted by $A^B(\zeta)$, $B^B(\zeta)$, and $D^{A(B)}(\zeta)$ there]. The X^A corresponds to the concentration of the A-component at the zeroth order in ϵ ($X^A = n^A_{H(0)}/n_{H(0)}$), so that only A^B, B^B, and $D^{A(B)}$ at $X^A = 1$ occur in Eq. (54). The definitions of these functions are summarized in Appendix D. Setting $n^B_{H(0)} = 0$ in Eqs. (B3b) and (B3c) (with $\alpha = B$) multiplied by $n^B_{H(0)}$ and using Eqs. (28) and (37) (note that $\hat{v}^\alpha_{3H(0)} = \hat{v}_{3H(0)} = 0$), we have

$$\hat{v}^B_{iH(1)} = \frac{1}{\hat{n}^B_{H(1)}} \int (\zeta_i - \hat{v}_{iH(0)}) \hat{F}^B_{H(2)} d^3\zeta, \tag{55a}$$

$$\hat{T}^B_{H(1)} = \frac{2\hat{m}^B}{3\hat{n}^B_{H(1)}} \int (\zeta_j - \hat{v}_{jH(0)})^2 \hat{F}^B_{H(2)} d^3\zeta - \left(\frac{\hat{n}^B_{H(2)}}{\hat{n}^B_{H(1)}}\right) \hat{T}_{H(0)}. \tag{55b}$$

Substitution of Eq. (54) into Eq. (55a) yields

$$\hat{v}^B_{iH(1)} - \hat{v}_{iH(1)} = \left(\hat{\Delta}^*_{BA} \frac{\hat{T}^{1/2}_{H(0)}}{\hat{\rho}^B_{H(1)}} \frac{1}{\hat{p}_{H(0)}} \frac{d\hat{p}^B_{H(1)}}{dy} - \hat{D}^*_{TB} \frac{\hat{T}^{1/2}_{H(0)}}{\hat{p}_{H(0)}} \frac{d\hat{T}_{H(0)}}{dy} \right) \delta_{i1}, \tag{56}$$

where δ_{ij} is the Kronecker delta, $\hat{\Delta}^*_{BA}$ and $\hat{D}^*_{TB}$, which are constants depending on $\hat{m}^B$ and $\hat{d}^B$, are defined in Appendix D, and their numerical values are given there (Table III). On the other hand, Eq. (55b) with Eq. (54) gives

$$\hat{T}^B_{H(1)} = \hat{T}_{H(1)}. \tag{57}$$

If we substitute the explicit form of $\hat{F}^B_{H(2)}$ [Eq. (54)] into the condition $\mathcal{S}^B_{0(2)}$ in Eq. (45), we obtain

$$\frac{d}{dy}\left(\hat{n}^B_{H(1)} \hat{v}^B_{1H(1)}\right) = 0. \tag{58}$$

To summarize, Eqs. (48), (52a)–(52d), (56) (with $i = 1$), and (58) with the relations $\hat{p}_{H(0)} = \hat{n}_{H(0)} \hat{T}_{H(0)}$ [Eq. (B1f)], $\hat{\rho}^B_{H(1)} = \hat{m}^B \hat{n}^B_{H(1)}$ [Eq. (B2a)], and $\hat{p}^B_{H(1)} = \hat{n}^B_{H(1)} \hat{T}_{H(0)}$ [Eq. (B2d) with $\hat{n}^B_{H(0)} = 0$] form a set of fluid-dynamic equations for $\hat{p}_{H(0)}$, $\hat{n}_{H(0)}$, $\hat{T}_{H(0)}$, $\hat{v}_{2H(0)}$, $\hat{v}_{1H(1)}$, $\hat{p}^A_{H(1)}$, $\hat{p}^B_{H(1)}$, $\hat{n}^B_{H(1)}$, $\hat{\rho}^B_{H(1)}$, and $\hat{v}^B_{1H(1)}$. If we proceed to the next order, the conditions $\mathcal{S}^A_{0(2)}$, $\mathcal{S}^A_{i(2)}$, $\mathcal{S}^A_{4(2)}$, and $\mathcal{S}^B_{0(3)}$ give the fluid-dynamic-type equations for the quantities of the next order and do not add any constraint for the leading-order quantities listed above.

4. BOUNDARY CONDITIONS FOR FLUID-DYNAMIC EQUATIONS

4.1. Knudsen Layers

In Sec. 3, we have derived the slowly-varying solution and associated fluid-dynamic equations, regardless of the boundary condition on the condensed phase. The solution is meaningful only when it can be made to satisfy the boundary condition or can be matched with another solution that satisfies the condition. To study this problem, we first expand the boundary conditions (10a) and (10b) in ϵ. Let us suppose that the original (dimensionless) velocity distribution functions $\hat{F}^A$ and $\hat{F}^B$ are expanded in ϵ as

$$\hat{F}^\alpha = \hat{F}^\alpha_{(0)} + \hat{F}^\alpha_{(1)}\epsilon + \hat{F}^\alpha_{(2)}\epsilon^2 + \cdots, \quad (\alpha = A,\ B). \tag{59}$$

Then, Eqs. (10a) and (10b) lead to the following boundary conditions for the component functions $\hat{F}^\alpha_{(m)}$ of the expansion (59):

$$\hat{F}^\alpha_{(m)} = \hat{F}^\alpha_{w(m)}, \qquad (\zeta_1 > 0,\ x_1 = 0), \tag{60}$$

with $m = 0,\ 1,\ 2,\ \ldots$, where

$$\hat{F}^A_{w(0)} = \pi^{-3/2} \exp(-\zeta_j^2), \qquad \hat{F}^A_{w(m+1)} = 0, \tag{61a}$$

$$\hat{F}^B_{w(m)} = \left(\frac{\hat{m}^B}{\pi}\right)^{3/2} \hat{\sigma}^B_{w(m)} \exp(-\hat{m}^B \zeta_j^2), \tag{61b}$$

$$\hat{\sigma}^B_{w(m)} = -2(\pi \hat{m}^B)^{1/2} \int_{\zeta_1 < 0} \zeta_1 \hat{F}^B_{(m)}(x_1 = 0,\ \zeta_i)\, d^3\zeta. \tag{61c}$$

Now let us try to fit the slowly-varying solution (23) to the boundary condition (60). Equation (31) obviously satisfies Eq. (60) with $\alpha = B$ and $m = 0$. It is seen from Eqs. (30) and (35) that $\hat{F}^A_{H(0)}$ and $\hat{F}^B_{H(1)}$ satisfy, respectively, Eq. (60) with $(\alpha = A, m = 0)$ and that with $(\alpha = B, m = 1)$ if we assume that $\hat{n}_{H(0)}$, $\hat{T}_{H(0)}$, and $\hat{v}_{2H(0)}$ take the following values on the condensed phase:

$$\hat{n}_{H(0)} = 1, \qquad \hat{T}_{H(0)} = 1, \qquad \hat{v}_{2H(0)} = 0, \qquad \text{at } y = 0. \tag{62}$$

However, the higher-order terms $\hat{F}^\alpha_{H(m)}$ ($m \geq 1$ for $\alpha = A$ and $m \geq 2$ for $\alpha = B$) do not have enough freedom to satisfy the corresponding boundary conditions (60) ($m \geq 1$ for $\alpha = A$ and $m \geq 2$ for $\alpha = B$). For instance, in order to make $\hat{F}^A_{H(1)}$ [Eq. (49) with Eq. (51)] and $\hat{F}^B_{H(2)}$ [Eq. (54)] satisfy the corresponding boundary conditions, we have to impose $d\hat{T}_{H(0)}/dy = d\hat{v}_{2H(0)}/dy = d\hat{p}^B_{H(1)}/dy = 0$ in addition to $\hat{p}^A_{H(1)} = \hat{T}_{H(1)} = \hat{v}_{1H(1)} = \hat{v}_{2H(1)} = 0$ at $y = 0$. These conditions for $\hat{T}_{H(0)}$, $\hat{v}_{2H(0)}$, and $\hat{v}_{1H(1)}$ and Eq. (62) are too many in view of the form of Eqs. (52a), (52c), and (52d) and the fact that $\hat{T}_{H(0)}$, $\hat{v}_{2H(0)}$, and $\hat{v}_{1H(1)}$ should satisfy some (independent) conditions at infinity [see Eqs. (79) and (80)].

Therefore, we seek the solution satisfying the boundary condition in the form

$$\hat{F}^\alpha = \hat{F}^\alpha_H + \hat{F}^\alpha_K, \tag{63a}$$

$$\hat{F}^A_K = \hat{F}^A_{K(1)}\epsilon + \hat{F}^A_{K(2)}\epsilon^2 + \cdots, \tag{63b}$$

$$\hat{F}^B_K = \hat{F}^B_{K(2)}\epsilon^2 + \hat{F}^B_{K(3)}\epsilon^3 + \cdots, \tag{63c}$$

where $\hat{F}^\alpha_K$ is a correction to the slowly-varying solution in a layer with thickness of the order of the mean free path [or $x_1 < O(1)$] adjacent to the condensed phase (Knudsen layer) (cf. ref. 2). We assume that its length scale of variation is of the order of the mean free path, i.e., $\hat{F}^\alpha_K = \hat{F}^\alpha_K(x_1, \zeta_i)$ [or $\partial\hat{F}^\alpha_K/\partial x_1 = O(\hat{F}^\alpha_K)$] and that it vanishes rapidly as x_1 tends to infinity. The expansion of $\hat{F}^A_K$ (or $\hat{F}^B_K$) is started from $\hat{F}^A_{K(1)}$ (or $\hat{F}^B_{K(2)}$) because $\hat{F}^A_{H(0)}$ (or $\hat{F}^B_{H(1)}$) can be made to satisfy the boundary conditions with the choice (62) of the boundary values.

Corresponding to Eq. (63), the macroscopic variables h ($h = \hat{n}^\alpha$, $\hat{v}^\alpha_i$, $\hat{T}^\alpha$, $\hat{n}$, $\hat{v}_i$, $\hat{T}$, etc.) are expressed as

$$h = h_H + h_K, \tag{64a}$$

$$h_K = h_{K(1)}\epsilon + h_{K(2)}\epsilon^2 + \cdots. \tag{64b}$$

The relation between h_K and $\hat{F}^\alpha_K$ is obtained by inserting Eqs. (63) and (64) into Eqs. (13a)–(13f) and by taking into account the relation between h_H and $\hat{F}^\alpha_H$. The result is omitted for conciseness.

Substituting Eq. (63a) with Eqs. (23), (63b), and (63c) into the Boltzmann equation (8) and taking into account the fact that (i) $\hat{F}^\alpha_H$ is a solution of Eq. (8), and (ii) $\hat{F}^\alpha_H$ can be Taylor expanded in x_1 variable as

$$\hat{F}^A_H = \left(\hat{F}^A_{H(0)}\right)_b + \left[\left(\hat{F}^A_{H(1)}\right)_b + \left(\frac{\partial \hat{F}^A_{H(0)}}{\partial y}\right)_b x_1\right]\epsilon + \cdots, \tag{65a}$$

$$\hat{F}^B_H = \left(\hat{F}^B_{H(1)}\right)_b\epsilon + \left[\left(\hat{F}^B_{H(2)}\right)_b + \left(\frac{\partial \hat{F}^B_{H(1)}}{\partial y}\right)_b x_1\right]\epsilon^2 + \cdots, \tag{65b}$$

where $(\ )_b$ indicates the value at $y = 0$ (or $x_1 = 0$), we obtain the equations for $\hat{F}^\alpha_{K(m)}$. On the other hand, the boundary conditions for $\hat{F}^\alpha_{K(m)}$ on the condensed phase are derived from the requirement that $\hat{F}^\alpha_{(m)} = \hat{F}^\alpha_{H(m)} + \hat{F}^\alpha_{K(m)}$ satisfy Eq. (60) for $m \geq 1$ when $\alpha = A$ and for $m \geq 2$ when $\alpha = B$. Furthermore, the condition $\hat{F}^\alpha_{K(m)} \to 0$ as $x_1 \to \infty$ should be imposed.

Here, we give the explicit form of the equations and boundary conditions only for the leading terms: $\hat{F}^A_{K(1)}$ and $\hat{F}^B_{K(2)}$. We first let

$$\hat{F}^A_{K(1)} = E^A(\zeta)\phi^A, \quad \hat{F}^B_{K(2)} = (\hat{n}^B_{H(1)})_b E^B(\zeta)\phi^B, \tag{66}$$

where $E^A(\zeta)$ and $E^B(\zeta)$ are defined by Eq. (C3). Then, the equations for ϕ^A and ϕ^B are given by

$$\zeta_1 \frac{\partial \phi^A}{\partial x_1} = \widetilde{L}^{AA}(\phi^A, \phi^A), \tag{67a}$$

$$\zeta_1 \frac{\partial \phi^B}{\partial x_1} = \hat{K}^{AB}\widetilde{L}^{AB}(\phi^A, \phi^B), \tag{67b}$$

where $\widetilde{L}^{\beta\alpha}$ is the linearized collision operator defined by Eq. (C4). The corresponding boundary conditions on the condensed phase can be written in the following form:

$$\phi^A = -(\hat{p}^A_{H(1)})_b - 2(\hat{v}_{1H(1)})_b\zeta_1 - 2(\hat{v}_{2H(1)})_b\zeta_2 - (\hat{T}_{H(1)})_b\left(\zeta^2 - \frac{5}{2}\right) + \left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b \zeta_1 A(\zeta) + \left(\frac{d\hat{v}_{2H(0)}}{dy}\right)_b \zeta_1\zeta_2 B(\zeta), \qquad \text{for } \zeta_1 > 0 \quad \text{at } x_1 = 0, \tag{68a}$$

$$\phi^B = \kappa^B_w - 2\hat{m}^B(\hat{v}_{1H(1)})_b\zeta_1 - 2\hat{m}^B(\hat{v}_{2H(1)})_b\zeta_2 - (\hat{T}_{H(1)})_b\left(\hat{m}^B\zeta^2 - \frac{5}{2}\right) + \left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b \zeta_1 A^B(\zeta; X^A = 1) + \left(\frac{d\hat{v}_{2H(0)}}{dy}\right)_b \zeta_1\zeta_2 B^B(\zeta; X^A = 1) - \frac{1}{(\hat{\rho}^B_{H(1)})_b}\left(\frac{d\hat{p}^B_{H(1)}}{dy}\right)_b \zeta_1 D^{(A)B}(\zeta; X^A = 1), \qquad \text{for } \zeta_1 > 0 \quad \text{at } x_1 = 0, \tag{68b}$$

with

$$\kappa^B_w = -(\pi\hat{m}^B)^{1/2}(\hat{v}_{1H(1)})_b - \frac{1}{2}(\hat{T}_{H(1)})_b + (\pi\hat{m}^B)^{1/2}\left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b \hat{D}^*_{TB} - \frac{(\pi\hat{m}^B)^{1/2}}{(\hat{\rho}^B_{H(1)})_b}\left(\frac{d\hat{p}^B_{H(1)}}{dy}\right)_b \hat{\Delta}^*_{BA} - 2(\pi\hat{m}^B)^{1/2}\int_{\zeta_1<0} \zeta_1\phi^B E^B(\zeta) d^3\zeta. \tag{69}$$

On the other hand, the conditions at infinity are simply

$$\phi^A \to 0 \quad \text{(rapidly)}, \qquad \text{as } x_1 \to \infty, \tag{70a}$$

$$\phi^B \to 0 \quad \text{(rapidly)}, \qquad \text{as } x_1 \to \infty. \tag{70b}$$

Equations (67a), (68a), and (70a) form a half-space boundary-value problem of the linearized Boltzmann equation for a single-component gas, which is known as the Knudsen-layer problem. This problem has a unique solution only when the constants $(\hat{p}^A_{H(1)})_b$, $(\hat{T}_{H(1)})_b$, and $(\hat{v}_{2H(1)})_b$ are related with the constants $(\hat{v}_{1H(1)})_b$, $(d\hat{T}_{H(0)}/dy)_b$, and $(d\hat{v}_{2H(0)}/dy)_b$ in a special manner.(2) This is a consequence of the theorem first conjectured by Grad in 1969(29) but proved much later by Bardos *et al.*(30) for hard-sphere molecules. The theorem has also been proved for more general molecular models.(31,32) The relations among the constants mentioned above are given explicitly as(2)

$$\left(\hat{p}^A_{H(1)}\right)_b = C_4^*(\hat{v}_{1H(1)})_b + C_1\left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b, \tag{71a}$$

$$(\hat{T}_{H(1)})_b = d_4^*(\hat{v}_{1H(1)})_b + d_1\left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b, \tag{71b}$$

$$(\hat{v}_{2H(1)})_b = -k_0\left(\frac{d\hat{v}_{2H(0)}}{dy}\right)_b, \tag{71c}$$

where C_4^*, C_1, d_4^*, d_1, and k_0 are constants. Equation (71a) gives a boundary condition for the present fluid-dynamic equations. It should be mentioned that Eqs. (71b) and (71c) form a part of the boundary condition for the higher-order fluid-dynamic equations. The constants occurring in Eq. (71a) for the present hard-sphere molecules are given as [see Eq. (3.78) in ref. 2]

$$C_4^* = -2.1412, \qquad C_1 = 1.0947. \tag{72}$$

Suppose that the solution ϕ^A is known. Since $\widetilde{L}^{AB}(\phi^A, \phi^B) = \widetilde{L}^{AB}(0, \phi^B) + \widetilde{L}^{AB}(\phi^A, 0)$, and $\widetilde{L}^{AB}(0, \phi^B)$ is the linear collision operator for ϕ^B, Eqs. (67b), (68b), and (70b) form a half-space boundary-value problem of the linear Boltzmann equation with the inhomogeneous term $\hat{K}^{AB}\widetilde{L}^{AB}(\phi^A, 0)$. The half-space problems for the linear Boltzmann equation has been investigated mathematically (e.g., refs. 33–35). Let us now assume that κ_w^B in Eq. (68b) is an undetermined constant, putting aside Eq. (69). According to the theorem by Arkeryd and Nouri(35), the constant κ_w^B is uniquely determined without any constraint on the other constants $(\hat{v}_{1H(1)})_b$, $(\hat{v}_{2H(1)})_b$, ..., $(d\hat{p}^B_{H(1)}/dy)_b$ contained in Eq. (68b), and the relation

$$\int \zeta_1\phi^B E^B(\zeta)d^3\zeta = 0, \quad \text{for } x_1 \geq 0, \tag{73}$$

holds (in ref. 35, the theorem is proved for the linear Boltzmann equation without an inhomogeneous term, but the theorem holds also for the case with an inhomogeneous term satisfying $\int(\,\cdot\,)E^B d^3\zeta = 0$ (36)). From Eq. (68b) and Eq. (73) at $x_1 = 0$, we can calculate $\int_{\zeta_1<0}\zeta_1\phi^B E^B(\zeta)d^3\zeta$ at $x_1 = 0$ in terms of κ_w^B, $(\hat{v}_{1H(1)})_b$,

$(\hat{T}_{H(1)})_b$, $(d\hat{T}_{H(0)}/dy)_b$, and $(d\hat{p}^B_{H(1)}/dy)_b$. Using this result in Eq. (69) and taking into account Eqs. (56) and (62), we obtain

$$\left(\hat{v}^B_{1H(1)}\right)_b = 0. \tag{74}$$

That is, Eq. (69) is equivalent to Eq. (74).

In summary, the boundary conditions on the condensed phase for the fluid-dynamic equations are given by Eqs. (62), (71a), and (74). It follows from Eqs. (58) and (74) that

$$\hat{v}^B_{1H(1)} = 0, \quad (\text{for } y \geq 0), \tag{75}$$

holds. This conclusion is obvious from Eq. (18).

4.2. Conditions at Infinity

Finally we should mention the boundary conditions at infinity for the fluid-dynamic equations. We assume that only $\hat{p}_\infty$ among the parameters $\hat{p}_\infty$, $\hat{T}_\infty$, $\hat{v}_{2\infty}$, and Γ depends on ϵ and expand it as

$$\hat{p}_\infty = \hat{p}_{\infty(0)} + \hat{p}_{\infty(1)}\epsilon + \hat{p}_{\infty(2)}\epsilon^2 + \cdots . \tag{76}$$

This assumption does not lead to any contradiction and is consistent with the fact that we are trying to obtain the expansion of the function $\mathcal{F}_s$ in Eq. (20a) in $M_{n\infty}$ (or ϵ) when $M_{n\infty}$ is small. Here again, we consider the expansion (59) of the original velocity distribution functions $\hat{F}^A$ and $\hat{F}^B$. Then, from Eqs. (12a) and (12b), we have the following boundary conditions for the component functions $\hat{F}^\alpha_{(m)}$:

$$\hat{F}^\alpha_{(m)} \to \hat{F}^\alpha_{\infty(m)}, \qquad (x_1 \to \infty), \tag{77}$$

with $m = 0, 1, 2, \ldots$, where

$$\hat{F}^A_{\infty(0)} = \frac{\hat{p}_{\infty(0)}}{\pi^{3/2}\hat{T}_\infty^{5/2}} \exp\left(-\frac{\zeta_1^2 + (\zeta_2 - \hat{v}_{2\infty})^2 + \zeta_3^2}{\hat{T}_\infty}\right), \tag{78a}$$

$$\hat{F}^A_{\infty(1)} = \left(\frac{\hat{p}_{\infty(1)}}{\hat{p}_{\infty(0)}} - 2\frac{\zeta_1}{\hat{T}_\infty}\right)\hat{F}^A_{\infty(0)}, \tag{78b}$$

$$\cdots,$$

$$\hat{F}^B_{\infty(m)} = 0, \quad (m = 0, 1, \cdots). \tag{78c}$$

The slowly-varying solution $\hat{F}^\alpha_H$ should satisfy the condition (77). From Eqs. (30)

(with $\hat{n}_{H(0)} = \hat{p}_{H(0)}/\hat{T}_{H(0)}$) and (35), we have

$$(\hat{p}_{H(0)})_\infty = \hat{p}_{\infty(0)}, \quad (\hat{T}_{H(0)})_\infty = \hat{T}_\infty, \quad (\hat{v}_{2H(0)})_\infty = \hat{v}_{2\infty}, \quad (\hat{n}^B_{H(1)})_\infty = 0, \tag{79}$$

where $(\ )_\infty$ represents the limiting value at infinity. From Eqs. (49), (51), and (54), we obtain the following conditions:

$$\begin{aligned} &\left(\hat{p}^A_{H(1)}\right)_\infty = \hat{p}_{\infty(1)}, \quad \left(\hat{v}_{1H(1)}\right)_\infty = -1, \quad \left(\hat{v}_{2H(1)}\right)_\infty = 0, \\ &\left(\hat{T}_{H(1)}\right)_\infty = 0, \ \left(\hat{n}^B_{H(2)}\right)_\infty = 0, \end{aligned} \tag{80}$$

because Eq. (79) implies that $(d\hat{T}_{H(0)}/dy)_\infty = (d\hat{v}_{2H(0)}/dy)_\infty = (d\hat{n}^B_{H(1)}/dy)_\infty = 0$. Equation (79) and the first two conditions of Eq. (80) serve as the boundary conditions at infinity for our fluid-dynamic equations summarized at the end of the preceding section. The rest of Eq. (80) provides a part of the boundary conditions for the fluid-dynamic equations of the next order in ϵ.

5. SUMMARY OF THE FLUID-DYNAMIC EQUATIONS AND THEIR BOUNDARY CONDITIONS

In Secs. 3 and 4, we have derived the fluid-dynamic equations and boundary conditions for the macroscopic variables of the slowly-varying solution. Here, we summarize them for later convenience. The equations are

$$\frac{d\hat{p}_{H(0)}}{dy} = 0, \tag{81a}$$

$$\frac{d}{dy}\left(\hat{n}_{H(0)}\hat{v}_{1H(1)}\right) = 0, \tag{81b}$$

$$\frac{d}{dy}\left(\hat{n}_{H(0)}\hat{v}_{1H(1)}\hat{v}_{2H(0)} - \frac{\gamma_1}{2}\hat{T}^{1/2}_{H(0)}\frac{d\hat{v}_{2H(0)}}{dy}\right) = 0, \tag{81c}$$

$$\frac{d}{dy}\left[\hat{n}_{H(0)}\hat{v}_{1H(1)}\left(\frac{5}{2}\hat{T}_{H(0)} + \hat{v}^2_{2H(0)}\right) - \frac{5}{4}\gamma_2\hat{T}^{1/2}_{H(0)}\frac{d\hat{T}_{H(0)}}{dy} - \gamma_1\hat{T}^{1/2}_{H(0)}\hat{v}_{2H(0)}\frac{d\hat{v}_{2H(0)}}{dy}\right] = 0, \tag{81d}$$

$$\frac{d}{dy}(\hat{p}^A_{H(1)} + \hat{p}^B_{H(1)}) = 0, \tag{81e}$$

$$\hat{v}_{1H(1)} = -\hat{\Delta}^*_{BA}\frac{\hat{T}^{1/2}_{H(0)}}{\hat{m}^B\hat{n}^B_{H(1)}}\frac{1}{\hat{p}_{H(0)}}\frac{d\hat{p}^B_{H(1)}}{dy} + \hat{D}^*_{TB}\frac{\hat{T}^{1/2}_{H(0)}}{\hat{p}_{H(0)}}\frac{d\hat{T}_{H(0)}}{dy}, \tag{81f}$$

$$\hat{p}_{H(0)} = \hat{n}_{H(0)}\hat{T}_{H(0)}, \tag{81g}$$

$$\hat{p}^B_{H(1)} = \hat{n}^B_{H(1)}\hat{T}_{H(0)}, \tag{81h}$$

where $\gamma_1 = 1.270042$, $\gamma_2 = 1.922284$, and $\hat{\Delta}^*_{BA}$ and $\hat{D}^*_{TB}$ are given in Table III in Appendix D. The boundary conditions on the condensed phase ($y = 0$) are

$$\begin{aligned} &(\hat{n}_{H(0)})_b = 1, \qquad (\hat{T}_{H(0)})_b = 1, \qquad (\hat{v}_{2H(0)})_b = 0, \\ &(\hat{p}^A_{H(1)})_b = C_4^*(\hat{v}_{1H(1)})_b + C_1\left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b, \end{aligned} \tag{82}$$

where $C_4^* = -2.1412$ and $C_1 = 1.0947$, and those at infinity ($y \to \infty$) are

$$\begin{aligned} &(\hat{p}_{H(0)})_\infty = \hat{p}_{\infty(0)}, \qquad (\hat{T}_{H(0)})_\infty = \hat{T}_\infty, \qquad (\hat{v}_{2H(0)})_\infty = \hat{v}_{2\infty}, \\ &(\hat{n}^B_{H(1)})_\infty = 0, \qquad (\hat{p}^A_{H(1)})_\infty = \hat{p}_{\infty(1)}, \qquad (\hat{v}_{1H(1)})_\infty = -1. \end{aligned} \tag{83}$$

Here, Eq. (75) has been used.

6. SOLUTION TO FLUID-DYNAMIC SYSTEM

In this section, we solve the fluid-dynamic system summarized in Sec. 5 to obtain the macroscopic quantities of the slowly-varying solution.

First of all, we recall that the parameter Γ is defined by Eq. (17). If we use the expansion of $\hat{n}^B = \hat{n}^B_H + \hat{n}^B_K$ in Eq. (17) and note that $\hat{n}^B_H$ is the function of y [Eq. (21)], Γ is expanded as

$$\Gamma = \Gamma_{(0)} + \Gamma_{(1)}\epsilon + \Gamma_{(2)}\epsilon^2 + \cdots, \tag{84}$$

where

$$\Gamma_{(0)} = \int_0^\infty \hat{n}^B_{H(1)} dy, \tag{85a}$$

$$\Gamma_{(1)} = \int_0^\infty \hat{n}^B_{H(2)} dy, \tag{85b}$$

$$\Gamma_{(2)} = \int_0^\infty \hat{n}^B_{H(3)} dy + \int_0^\infty \hat{n}^B_{K(2)} dx_1, \tag{85c}$$

$$\cdots.$$

Since we can assume without loss of generality that Γ is independent of ϵ, we let $\Gamma_{(1)} = \Gamma_{(2)} = \Gamma_{(3)} = \cdots = 0$, so that

$$\Gamma = \Gamma_{(0)} = \int_0^\infty \hat{n}^B_{H(1)} dy. \tag{86}$$

The condition $\Gamma_{(1)} = \Gamma_{(2)} = \Gamma_{(3)} = \cdots = 0$ gives natural constraints for higher-order fluid-dynamic equations.

Let us go back to the fluid-dynamic equations. Equations (81a), (81g), and the first two conditions in Eq. (82) show that

$$\hat{p}_{H(0)} \equiv 1. \tag{87}$$

From this and the first condition in Eq. (83), we find that

$$\hat{p}_{\infty(0)} = 1. \tag{88}$$

Equations (81g) and (87) give $\hat{n}_{H(0)} = 1/\hat{T}_{H(0)}$. Therefore, from Eq. (81b) and the conditions at infinity (83), we have $\hat{n}_{H(0)}\hat{v}_{1H(1)} = (\hat{n}_{H(0)})_\infty(\hat{v}_{1H(1)})_\infty = -1/\hat{T}_\infty$. To summarize,

$$\hat{n}_{H(0)} = 1/\hat{T}_{H(0)}, \qquad \hat{v}_{1H(1)} = -\hat{T}_{H(0)}/\hat{T}_\infty. \tag{89}$$

The integration of Eqs. (81c) and (81d) with Eq. (89) leads to the following equations:

$$\frac{1}{\hat{T}_\infty}\widetilde{v}_{2H(0)} + \frac{1}{2}\gamma_1\hat{T}_{H(0)}^{1/2}\frac{d\widetilde{v}_{2H(0)}}{dy} = 0, \tag{90a}$$

$$\frac{1}{\hat{T}_\infty}\left(\frac{5}{2}\widetilde{T}_{H(0)} - \widetilde{v}_{2H(0)}^2\right) + \frac{5}{4}\gamma_2\hat{T}_{H(0)}^{1/2}\frac{d\widetilde{T}_{H(0)}}{dy} = 0, \tag{90b}$$

where

$$\widetilde{v}_{2H(0)} = \hat{v}_{2H(0)} - \hat{v}_{2\infty}, \qquad \widetilde{T}_{H(0)} = \hat{T}_{H(0)} - \hat{T}_\infty. \tag{91}$$

Here, the condition at infinity (79), together with the fact that $d\hat{v}_{2H(0)}/dy$ and $d\hat{T}_{H(0)}/dy$ vanish at infinity, is used to fix the two arbitrary constants contained in Eqs. (90a) and (90b).

Equation (81e) and the condition at infinity (83) give

$$\hat{p}_{H(1)}^A = \hat{p}_{\infty(1)} - \hat{p}_{H(1)}^B = \hat{p}_{\infty(1)} - \hat{n}_{H(1)}^B\hat{T}_{H(0)}. \tag{92}$$

On the other hand, using Eqs. (81h), (87), and (89) in Eq. (81f), we have

$$\frac{d}{dy}\left(\ln\hat{n}_{H(1)}^B\right) = \frac{\hat{m}^B}{\hat{\Delta}_{BA}^*\hat{T}_\infty}\hat{T}_{H(0)}^{-1/2} + \left(\frac{\hat{m}^B\hat{D}_{TB}^*}{\hat{\Delta}_{BA}^*} - 1\right)\frac{d\ln\hat{T}_{H(0)}}{dy}. \tag{93}$$

We will obtain the solution to the fluid-dynamic system separately in the following three cases:

- Case I: $\hat{v}_{2\infty} = 0$ and $\hat{T}_\infty = 1$,
- Case II: $\hat{v}_{2\infty} = 0$ and $\hat{T}_\infty \neq 1$,
- Case III: $\hat{v}_{2\infty} \neq 0$.

• **Case I** ($\hat{v}_{2\infty} = 0$ and $\hat{T}_\infty = 1$)

In this case,

$$\hat{v}_{2H(0)} = 0, \qquad \hat{T}_{H(0)} = 1, \tag{94}$$

is the obvious solution of Eqs. (81c) and (81d) satisfying the boundary conditions $(\hat{v}_{2H(0)})_b = 0$, $(\hat{T}_{H(0)})_b = 1$, $(\hat{v}_{2H(0)})_\infty = \hat{v}_{2\infty} = 0$, and $(\hat{T}_{H(0)})_\infty = \hat{T}_\infty = 1$. It follows from Eq. (89) that

$$\hat{n}_{H(0)} = 1, \qquad \hat{v}_{1H(1)} = -1. \tag{95}$$

Then, Eq. (93) with the condition (86) gives the following $\hat{n}^B_{H(1)}$ that satisfies the condition at infinity $(\hat{n}^B_{H(1)})_\infty = 0$:

$$\hat{n}^B_{H(1)} = -\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}}\Gamma \exp\left(\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}} y\right). \tag{96}$$

(Note that $\hat{\Delta}^*_{BA}$ is negative, as shown in Appendix D.) If we substitute Eq. (92) with Eq. (96) into the last condition in Eq. (82), we have

$$\hat{p}_{\infty(1)} = -\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}}\Gamma - C^*_4. \tag{97}$$

In consequence, we obtain $\hat{p}^A_{H(1)}$ in the following form:

$$\hat{p}^A_{H(1)} = -\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}}\Gamma \left[1 - \exp\left(\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}} y\right)\right] - C^*_4. \tag{98}$$

- **Case II** ($\hat{v}_{2\infty} = 0$ and $\hat{T}_\infty \neq 1$)

  In this case,

$$\hat{v}_{2H(0)} = 0, \tag{99}$$

is the obvious solution of Eq. (81c) satisfying the boundary conditions $(\hat{v}_{2H(0)})_b = 0$ and $(\hat{v}_{2H(0)})_\infty = \hat{v}_{2\infty} = 0$. Then, Eq. (81d) is transformed into

$$\hat{T}^{1/2}_{H(0)} \frac{d\hat{T}_{H(0)}}{dy} = \left(\hat{T}^{1/2}_{H(0)} \frac{d\hat{T}_{H(0)}}{dy}\right)_b \exp\left(-\frac{2}{\gamma_2 \hat{T}_\infty}\int_0^y \frac{dy}{\hat{T}^{1/2}_{H(0)}}\right). \tag{100}$$

Because of $(\hat{T}_{H(0)})_b = 1$, Eq. (100) indicates that $\hat{T}_{H(0)}$ is monotonic in y, or $\hat{T}_{H(0)} = \text{const}$. But, the latter case is possible only when $\hat{T}_\infty = 1$, which is the Case I. Therefore, $\hat{T}_{H(0)}$ is monotonic in the present Case II. In consequence, Eq. (90b) with $\widetilde{v}_{2H(0)} = 0$, i.e.,

$$\frac{d\hat{T}_{H(0)}}{dy} + \frac{2}{\gamma_2 \hat{T}_\infty}\frac{\hat{T}_{H(0)} - \hat{T}_\infty}{\hat{T}^{1/2}_{H(0)}} = 0, \tag{101}$$

with the condition $(\hat{T}_{H(0)})_b = 1$ is solved implicitly as

$$y = -\frac{\gamma_2}{2}\hat{T}_\infty \int_1^{\hat{T}_{H(0)}} \frac{\sqrt{s}}{s - \hat{T}_\infty} ds$$
$$= -\gamma_2 \hat{T}_\infty \left[\sqrt{\hat{T}_{H(0)}} - 1 + \frac{\sqrt{\hat{T}_\infty}}{2} \ln \left| \frac{\left(\sqrt{\hat{T}_{H(0)}} - \sqrt{\hat{T}_\infty}\right)\left(1 + \sqrt{\hat{T}_\infty}\right)}{\left(\sqrt{\hat{T}_{H(0)}} + \sqrt{\hat{T}_\infty}\right)\left(1 - \sqrt{\hat{T}_\infty}\right)} \right| \right]. \quad (102)$$

It is seen that Eq. (102) verifies the condition at infinity $(\hat{T}_{H(0)})_\infty = \hat{T}_\infty$. With this $\hat{T}_{H(0)}$, the solution $\hat{n}_{H(0)}$ and $\hat{v}_{1H(1)}$ are obtained from Eq. (89).

Equation (101) is transformed as

$$\hat{T}_{H(0)}^{-1/2} = -\frac{\gamma_2}{2}\hat{T}_\infty \frac{d}{dy} \ln |\hat{T}_{H(0)} - \hat{T}_\infty|. \quad (103)$$

By substituting this in the first term on the right-hand side of Eq. (93) and integrating it, we obtain

$$\hat{n}^B_{H(1)} = A_{II}(\hat{T}_{H(0)})^{(\hat{m}^B \hat{D}^*_{TB}/\hat{\Delta}^*_{BA})-1} \left| \hat{T}_{H(0)} - \hat{T}_\infty \right|^{-(\gamma_2/2)(\hat{m}^B/\hat{\Delta}^*_{BA})}, \quad (104)$$

where A_{II} is an arbitrary constant, which should be related to Γ. That is, we insert Eq. (104) in Eq. (86) and change the integration variable from y to $\hat{T}_{H(0)}$ with the help of Eq. (102) and the monotonicity of $\hat{T}_{H(0)}$ in y ($y \in [0, \infty)$ corresponds to $\hat{T}_{H(0)} \in [1, \hat{T}_\infty]$). With further change of the integration variable from $\hat{T}_{H(0)}$ to t defined by $\hat{T}_{H(0)} - \hat{T}_\infty = (1 - \hat{T}_\infty)t$, we obtain the following relation:

$$\Gamma = (\gamma_2 \hat{T}_\infty/2)|1 - \hat{T}_\infty|^{-(\gamma_2/2)(\hat{m}^B/\hat{\Delta}^*_{BA})} G(\hat{T}_\infty) A_{II}, \quad (105)$$

where

$$G(\hat{T}_\infty) = \int_0^1 t^{-\frac{\gamma_2}{2}\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}}-1} \left[\hat{T}_\infty + (1 - \hat{T}_\infty)t\right]^{\frac{\hat{m}^B \hat{D}^*_{TB}}{\hat{\Delta}^*_{BA}} - \frac{1}{2}} dt. \quad (106)$$

Therefore, $\hat{n}^B_{H(1)}$ is obtained as

$$\hat{n}^B_{H(1)} = \frac{2}{\gamma_2} \frac{\Gamma}{\hat{T}_\infty G(\hat{T}_\infty)} (\hat{T}_{H(0)})^{\frac{\hat{m}^B \hat{D}^*_{TB}}{\hat{\Delta}^*_{BA}} - 1} \left(\frac{|\hat{T}_{H(0)} - \hat{T}_\infty|}{|1 - \hat{T}_\infty|} \right)^{-\frac{\gamma_2}{2}\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}}}, \quad (107)$$

where $\hat{T}_{H(0)}$ is given by Eq. (102). If we substitute Eq. (92) into the last condition of Eq. (82) and note that

$$(\hat{v}_{1H(1)})_b = -\frac{1}{\hat{T}_\infty}, \qquad \left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b = \frac{2}{\gamma_2}\frac{(\hat{T}_\infty - 1)}{\hat{T}_\infty},$$

$$(\hat{n}^B_{H(1)})_b = \frac{2}{\gamma_2}\frac{\Gamma}{\hat{T}_\infty G(\hat{T}_\infty)}, \tag{108}$$

which follow from Eqs. (89), (101), and (107), respectively, then we obtain the following expression of $\hat{p}_{\infty(1)}$:

$$\hat{p}_{\infty(1)} = \frac{1}{\hat{T}_\infty}\left[\frac{2}{\gamma_2}\frac{\Gamma}{G(\hat{T}_\infty)} + \frac{2}{\gamma_2}C_1(\hat{T}_\infty - 1) - C_4^*\right]. \tag{109}$$

Finally, $\hat{p}^A_{H(1)}$ is given by Eq. (92), where $\hat{p}_{\infty(1)}$, $\hat{n}^B_{H(1)}$, and $\hat{T}_{H(0)}$ have been obtained in Eqs. (109), (107), and (102).

- **Case III** ($\hat{v}_{2\infty} \neq 0$)

With Eq. (89), we can transform Eq. (81c) into the following form:

$$\hat{T}^{1/2}_{H(0)}\frac{d\hat{v}_{2H(0)}}{dy} = \left(\hat{T}^{1/2}_{H(0)}\frac{d\hat{v}_{2H(0)}}{dy}\right)_b \exp\left(-\frac{2}{\gamma_1\hat{T}_\infty}\int_0^y \frac{dy}{\hat{T}^{1/2}_{H(0)}}\right). \tag{110}$$

Since $(\hat{T}_{H(0)})_b = 1$, this indicates that $\hat{v}_{2H(0)}$ is monotonic in y, or $\hat{v}_{2H(0)} = \text{const.}$ But, the latter case, which is possible only when $\hat{v}_{2H(0)} = \hat{v}_{2\infty} = 0$ because of Eqs. (82) and (83), is the Case II. Thanks to the monotonicity of $\hat{v}_{2H(0)}$ [or $\widetilde{v}_{2H(0)}$ in Eq. (91)] in y, we can regard $\hat{T}_{H(0)}$ and $\widetilde{T}_{H(0)}$ as functions of $\hat{v}_{2H(0)}$ or $\widetilde{v}_{2H(0)}$ ($y \in [0,\ \infty)$ corresponds to $\hat{v}_{2H(0)} \in [0,\ \hat{v}_{2\infty}]$ or $\widetilde{v}_{2H(0)} \in [-\hat{v}_{2\infty},\ 0]$). Then, from Eqs. (90a) and (90b), we obtain the following differential equation for $\widetilde{T}_{H(0)}$:

$$\frac{d\widetilde{T}_{H(0)}}{d\widetilde{v}_{2H(0)}} - \frac{\gamma_1}{\gamma_2}\left(\frac{\widetilde{T}_{H(0)}}{\widetilde{v}_{2H(0)}} - \frac{2}{5}\widetilde{v}_{2H(0)}\right) = 0. \tag{111}$$

The condition for Eq. (111), corresponding to $(\hat{T}_{H(0)})_b = 1$, is

$$\widetilde{T}_{H(0)} = 1 - \hat{T}_\infty, \quad \text{at } \widetilde{v}_{2H(0)} = -\hat{v}_{2\infty}. \tag{112}$$

Equation (111) with the condition (112) is solved readily and gives the following $\hat{T}_{H(0)}$ $(= \widetilde{T}_{H(0)} + \hat{T}_\infty)$ as a function of $\widetilde{v}_{2H(0)}$ ($\widetilde{v}_{2H(0)} \leq 0$):

$$\hat{T}_{H(0)} = \hat{T}_\infty - \frac{2}{5}\frac{\gamma_1}{2\gamma_2 - \gamma_1}\widetilde{v}^2_{2H(0)} + \left(1 - \hat{T}_\infty + \frac{2}{5}\frac{\gamma_1}{2\gamma_2 - \gamma_1}\hat{v}^2_{2\infty}\right)\left(\frac{-\widetilde{v}_{2H(0)}}{\hat{v}_{2\infty}}\right)^{\frac{\gamma_1}{\gamma_2}}. \tag{113}$$

Then, Eq. (90a) with the condition

$$\widetilde{v}_{2H(0)} = -\hat{v}_{2\infty}, \qquad \text{at } y = 0, \tag{114}$$

is solved implicitly as

$$y = -\frac{\gamma_1 \hat{T}_\infty}{2} \int_{-\hat{v}_{2\infty}}^{\widetilde{v}_{2H(0)}} [\hat{T}_{H(0)}(\widetilde{v}_{2H(0)} = s)]^{1/2} \frac{ds}{s}. \tag{115}$$

To summarize, $\hat{T}_{H(0)}$ (in terms of $\hat{v}_{2H(0)}$) and $\hat{v}_{2H(0)}$ are, respectively, obtained as

$$\hat{T}_{H(0)} = \hat{T}_\infty - \frac{2}{5}\frac{\gamma_1}{2\gamma_2 - \gamma_1}(\hat{v}_{2H(0)} - \hat{v}_{2\infty})^2 + \left(1 - \hat{T}_\infty + \frac{2}{5}\frac{\gamma_1}{2\gamma_2 - \gamma_1}\hat{v}_{2\infty}^2\right)\left(\frac{\hat{v}_{2\infty} - \hat{v}_{2H0}}{\hat{v}_{2\infty}}\right)^{\frac{\gamma_1}{\gamma_2}}, \tag{116}$$

$$y = -\frac{\gamma_1 \hat{T}_\infty}{2} \int_0^{\hat{v}_{2H(0)}} [\hat{T}_{H(0)}(\hat{v}_{2H(0)} = t)]^{1/2} \frac{dt}{t - \hat{v}_{2\infty}}. \tag{117}$$

These solutions satisfy the boundary condition at infinity: $(\hat{T}_{H(0)})_\infty = \hat{T}_\infty$ and $(\hat{v}_{2H(0)})_\infty = \hat{v}_{2\infty}$ [Eq. (83)].

Equation (90a) is transformed into

$$\hat{T}_{H(0)}^{-1/2} = -\frac{\gamma_1}{2}\hat{T}_\infty \frac{d}{dy}\ln(-\widetilde{v}_{2H(0)}), \tag{118}$$

which is substituted in the first term on the right-hand side of Eq. (93). Then the resulting equation is integrated readily and gives

$$\hat{n}_{H(1)}^B = A_{III}(\hat{T}_{H(0)})^{(\hat{m}^B \hat{D}_{TB}^*/\hat{\Delta}_{BA}^*) - 1}(-\widetilde{v}_{2H(0)})^{-(\gamma_1/2)(\hat{m}^B/\hat{\Delta}_{BA}^*)}, \tag{119}$$

where A_{III} is an arbitrary constant, which is to be related to the parameter Γ by Eq. (86). That is, we substitute Eq. (119) into Eq. (86), change the integration variable from y to $\widetilde{v}_{2H(0)}$ with the help of Eq. (115), and introduce the new integration variable t by $\widetilde{v}_{2H(0)} = -\hat{v}_{2\infty}t$. Then, we obtain the following expression of Γ:

$$\Gamma = (\gamma_1 \hat{T}_\infty/2)(\hat{v}_{2\infty})^{-(\gamma_1/2)(\hat{m}^B/\hat{\Delta}_{BA}^*)} H_0(\hat{T}_\infty, \hat{v}_{2\infty}) A_{III}, \tag{120}$$

where

$$H_j(\hat{T}_\infty, \hat{v}_{2\infty}) = \int_0^1 t^{-\frac{\gamma_1}{2}\frac{\hat{m}^B}{\hat{\Delta}_{BA}^*} + j - 1}\left[\hat{T}_\infty - \frac{2}{5}\frac{\gamma_1}{2\gamma_2 - \gamma_1}\hat{v}_{2\infty}^2 t^2 + \left(1 - \hat{T}_\infty + \frac{2}{5}\frac{\gamma_1}{2\gamma_2 - \gamma_1}\hat{v}_{2\infty}^2\right) t^{\frac{\gamma_1}{\gamma_2}}\right]^{\frac{\hat{m}^B \hat{D}_{TB}^*}{\hat{\Delta}_{BA}^*} - \frac{1}{2}} dt, \tag{121}$$

with $j = 0, 1$ [H_1 will appear in Eq. (134)]. Therefore, the solution $\hat{n}^B_{H(1)}$ is obtained as

$$\hat{n}^B_{H(1)} = \frac{2}{\gamma_1}\frac{\Gamma}{\hat{T}_\infty H_0(\hat{T}_\infty, \hat{v}_{2\infty})}(\hat{T}_{H(0)})^{\frac{\hat{m}^B \hat{D}^*_{TB}}{\hat{\Delta}^*_{BA}}-1}\left(\frac{\hat{v}_{2\infty} - \hat{v}_{2H(0)}}{\hat{v}_{2\infty}}\right)^{-\frac{\gamma_1}{2}\frac{\hat{m}^B}{\hat{\Delta}^*_{BA}}}, \quad (122)$$

where $\hat{T}_{H(0)}$ and $\hat{v}_{2H(0)}$ are given by Eqs. (116) and (117), respectively.

We now note that the following boundary values are obtained from Eqs. (89), (90b), and (122):

$$(\hat{v}_{1H(1)})_b = -\frac{1}{\hat{T}_\infty}, \qquad \left(\frac{d\hat{T}_{H(0)}}{dy}\right)_b = \frac{2}{\gamma_2 \hat{T}_\infty}\left(\hat{T}_\infty - 1 + \frac{2}{5}\hat{v}^2_{2\infty}\right),$$
$$(\hat{n}^B_{H(1)})_b = \frac{2}{\gamma_1}\frac{\Gamma}{\hat{T}_\infty H_0(\hat{T}_\infty, \hat{v}_{2\infty})}. \quad (123)$$

Substituting Eq. (92) into the last condition of Eq. (82) and using Eq. (123), we obtain $\hat{p}_{\infty(1)}$ in the following form:

$$\hat{p}_{\infty(1)} = \frac{1}{\hat{T}_\infty}\left[\frac{2}{\gamma_1}\frac{\Gamma}{H_0(\hat{T}_\infty, \hat{v}_{2\infty})} + \frac{2C_1}{\gamma_2}\left(\hat{T}_\infty - 1 + \frac{2}{5}\hat{v}^2_{2\infty}\right) - C^*_4\right]. \quad (124)$$

Finally, with $\hat{p}_{\infty(1)}$, $\hat{n}^B_{H(1)}$, and $\hat{T}_{H(0)}$ given by Eqs. (124), (122), and (116), $\hat{p}^A_{H(1)}$ is given by Eq. (92).

In this way, the macroscopic quantities $\hat{p}_{H(0)}$, $\hat{v}_{2H(0)}$, $\hat{v}_{1H(1)}$, $\hat{T}_{H(0)}$, $\hat{p}^A_{H(1)}$, and $\hat{n}^B_{H(1)}$ are determined for specified $\hat{v}_{2\infty}$, $\hat{T}_\infty$, and Γ. The quantities $\hat{p}_{\infty(0)}$ and $\hat{p}_{\infty(1)}$ at infinity are not at our disposal but are determined from $\hat{v}_{2\infty}$, $\hat{T}_\infty$, and Γ. It should be noted that the flow velocity $\hat{v}_{2H(0)}$ and $\hat{v}_{1H(1)}$ and the temperature $\hat{T}_{H(0)}$ of the mixture do not depend on Γ, so that they are the same as in the case of a single-component vapor ($\Gamma = 0$). In fact, the results of Cases I and II with $\Gamma = 0$ have been obtained by Sone[16,2].

We observe that

$$\lim_{\hat{v}_{2\infty}\to 0} H_0(\hat{T}_\infty,\ \hat{v}_{2\infty}) = (\gamma_2/\gamma_1)G(\hat{T}_\infty), \quad (125a)$$

$$\lim_{\hat{T}_\infty\to 1,\ \hat{v}_{2\infty}\to 0} H_0(\hat{T}_\infty,\ \hat{v}_{2\infty}) = (\gamma_2/\gamma_1)\lim_{\hat{T}_\infty\to 1} G(\hat{T}_\infty) = -(\gamma_1/2)(\hat{\Delta}^*_{BA}/\hat{m}_B). \quad (125b)$$

Therefore, the results for $\hat{p}_{\infty 1}$ in the three cases, Eqs. (97), (109), and (124), can be unified in Eq. (124) by defining $H_0(\hat{T}_\infty,\ 0)$ and $H_0(1,\ 0)$ as the corresponding limiting values.

7. PARAMETER RELATION

In Sec. 6, we have solved the fluid-dynamic system to obtain the macroscopic quantities of the slowly-varying solution. As a result, we have obtained the expressions of $\hat{p}_{\infty(0)}$ and $\hat{p}_{\infty(1)}$ that are given by Eqs. (88) and (124). With these expressions, Eq. (76) gives the desired relation among parameters. We summarize the result using the parameters appearing in Eq. (20a). That is, the relation (20a) for small $M_{n\infty}$ is given as follows:

$$\frac{p_\infty}{p_w} = 1 + \left(\frac{5}{6}\right)^{1/2}\left(\frac{T_\infty}{T_w}\right)^{-1/2}\left\{\frac{2}{\gamma_1}\left(\frac{T_\infty}{T_w}\right)^{-\frac{m^B}{m^A}\frac{\hat{D}^*_{TB}}{\hat{\Delta}^*_{BA}}+\frac{1}{2}}\frac{\Gamma}{\mathcal{H}_0(T_\infty/T_w,\ M_{t\infty})}\right.$$
$$\left.+\frac{2C_1}{\gamma_2}\left[\frac{T_\infty}{T_w}\left(\frac{M_{t\infty}^2}{3}+1\right)-1\right]-C_4^*\right\}M_{n\infty}+O(M_{n\infty}^2), \tag{126}$$

where

$$\mathcal{H}_j\left(\frac{T_\infty}{T_w},\ M_{t\infty}\right)$$
$$=\left(\frac{T_\infty}{T_w}\right)^{-\frac{m^B}{m^A}\frac{\hat{D}^*_{TB}}{\hat{\Delta}^*_{BA}}+\frac{1}{2}} H_j\left(\frac{T_\infty}{T_w},\ \left(\frac{5}{6}\right)^{1/2}\left(\frac{T_\infty}{T_w}\right)^{1/2}M_{t\infty}\right)$$
$$=\int_0^1 t^{-\frac{\gamma_1}{2}\frac{m^B}{m^A}\frac{1}{\hat{\Delta}^*_{BA}}+j-1}\left[1-\frac{1}{3}\frac{\gamma_1}{2\gamma_2-\gamma_1}M_{t\infty}^2t^2\right.$$
$$\left.+\left(\frac{T_w}{T_\infty}-1+\frac{1}{3}\frac{\gamma_1}{2\gamma_2-\gamma_1}M_{t\infty}^2\right)t^{\frac{\gamma_1}{\gamma_2}}\right]^{\frac{m^B}{m^A}\frac{\hat{D}^*_{TB}}{\hat{\Delta}^*_{BA}}-\frac{1}{2}}dt, \tag{127}$$

with $j = 0,\ 1$ [$\mathcal{H}_1$ will appear in Eq. (134)]. Here, $\hat{m}^B$ has also been replaced by the original m^B/m^A. Note that $\hat{\Delta}^*_{BA}$, $\hat{D}^*_{TB}$, and thus $\mathcal{H}_j$ depend on m^B/m^A and d^B/d^A.

8. PARTICLE-FLOW RATE OF THE NONCONDENSABLE GAS

When the vapor flow at infinity has a tangential component ($\hat{v}_{2\infty} \neq 0$, or $M_{t\infty} \neq 0$), there is a macroscopic flow of the noncondensable gas along the condensed phase. Let us denote by $\mathcal{N}_f$ the total particle-flow rate (per unit width in X_3 and per unit time) of the noncondensable gas in the X_2 direction and by $\hat{\mathcal{N}}_f$

its dimensionless counterpart, which are defined respectively as

$$\mathcal{N}_f = \int_0^\infty n^B v_2^B \, dX_1, \tag{128a}$$

$$\hat{\mathcal{N}}_f = \frac{2}{\sqrt{\pi}} \frac{\mathcal{N}_f}{n_\infty l_\infty (2kT_\infty/m^A)^{1/2}}. \tag{128b}$$

Let us consider the case of arbitrary $M_{n\infty}$. The solution of the original half-space problem is determined by specifying the four parameters, $M_{n\infty}$, $M_{t\infty}$, T_∞/T_w, and Γ, when $M_{n\infty} < 1$ [cf. Eq. (20a)], and five parameters, $M_{n\infty}$, $M_{t\infty}$, T_∞/T_w, p_∞/p_w, and Γ satisfying Eq. (20b) or (20c), when $M_{n\infty} \geq 1$. Since $\hat{\mathcal{N}}_f$ is determined uniquely by the solution, it may be considered as a function of the parameters listed above, i.e.,

$$\hat{\mathcal{N}}_f = G_s \left(M_{n\infty}, M_{t\infty}, \frac{T_\infty}{T_w}, \Gamma \right), \qquad (M_{n\infty} < 1), \tag{129a}$$

$$\hat{\mathcal{N}}_f = G_b \left(M_{n\infty}, M_{t\infty}, \frac{T_\infty}{T_w}, \frac{p_\infty}{p_w}, \Gamma \right), \qquad (M_{n\infty} \geq 1). \tag{129b}$$

Here, we try to obtain the leading-order term of Eq. (129a) when $M_{n\infty} \ll 1$.

Since $n_w l_w = n_\infty l_\infty$ holds for hard-sphere molecules, $\hat{\mathcal{N}}_f$ can be expressed as

$$\hat{\mathcal{N}}_f = \hat{T}_\infty^{-1/2} \int_0^\infty \hat{n}^B \hat{v}_2^B \, dx_1. \tag{130}$$

By substituting the expansions $\hat{n}^B = \hat{n}^B_{H(1)}\epsilon + (\hat{n}^B_{H(2)} + \hat{n}^B_{K(2)})\epsilon^2 + \cdots$ and $\hat{v}_2^B = \hat{v}^B_{2H(0)} + (\hat{v}^B_{2H(1)} + \hat{v}^B_{2K(1)})\epsilon + \cdots$ into Eq. (130) and taking into account the property of the slowly-varying solution, we obtain the following expansion of $\hat{\mathcal{N}}_f$ in ϵ:

$$\hat{\mathcal{N}}_f = \hat{\mathcal{N}}_{f(0)} + \hat{\mathcal{N}}_{f(1)}\epsilon + \hat{\mathcal{N}}_{f(2)}\epsilon^2 + \cdots, \tag{131}$$

where

$$\hat{\mathcal{N}}_{f(0)} = \hat{T}_\infty^{-1/2} \int_0^\infty \hat{n}^B_{H(1)} \hat{v}^B_{2H(0)} \, dy, \tag{132}$$

etc. (Note that $\hat{v}^B_{2K(1)}$ is not zero but is defined in terms of $\hat{F}^B_{K(2)}$.) Here, we recall that $\hat{v}^B_{2H(0)} = \hat{v}_{2H(0)} = \widetilde{v}_{2H(0)} + \hat{v}_{2\infty}$ [Eqs. (37) and (91)] and change the integration variable from y to $\widetilde{v}_{2H(0)}$ with the help of Eq. (115). Then we have

$$\hat{\mathcal{N}}_{f(0)} = -\frac{\gamma_1}{2} \hat{T}_\infty^{1/2} \int_{-\hat{v}_{2\infty}}^0 \hat{n}^B_{H(1)} \left(\widetilde{v}_{2H(0)} + \hat{v}_{2\infty} \right) \frac{\hat{T}^{1/2}_{H(0)}}{\widetilde{v}_{2H(0)}} \, d\widetilde{v}_{2H(0)}. \tag{133}$$

Using Eq. (113) for $\hat{T}_{H(0)}$ and Eq. (122) (with $\hat{v}_{2\infty} - \hat{v}_{2H(0)}$ replaced with $-\widetilde{v}_{2H(0)}$) for $\hat{n}^B_{H(1)}$ and introducing the new integration variable t by $\widetilde{v}_{2H(0)} = -\hat{v}_{2\infty}t$ as in the derivation of Eq. (120), we finally obtain the following expression for $\hat{\mathcal{N}}_f$:

$$\hat{\mathcal{N}}_f = \Gamma \hat{T}_\infty^{-1/2}\hat{v}_{2\infty}\left[1 - \frac{H_1(\hat{T}_\infty, \hat{v}_{2\infty})}{H_0(\hat{T}_\infty, \hat{v}_{2\infty})}\right] + O(\epsilon)$$

$$= \left(\frac{5}{6}\right)^{1/2}\Gamma M_{t\infty}\left[1 - \frac{\mathcal{H}_1(T_\infty/T_w, M_{t\infty})}{\mathcal{H}_0(T_\infty/T_w, M_{t\infty})}\right] + O(M_{n\infty}), \tag{134}$$

where H_j and $\mathcal{H}_j$ are defined by Eqs. (121) and (127), respectively.

9. SOME NUMERICAL RESULTS

In Secs. 6–8, we have obtained the analytical solution to the fluid-dynamic system and explicit expressions of the parameter relation and of the particle-flow rate of the noncondensable gas. The results contain some integrals to be evaluated. For example, the parameter relation (126) and the total particle-flow rate (134) contain, respectively, $\mathcal{H}_0$ and $\mathcal{H}_1$ defined by Eq. (127). With the values of $\hat{\Delta}^*_{BA}$ and $\hat{D}^*_{TB}$ given in Appendix D (Table III), the integrals in Eq. (127) are integrated numerically by means of the Simpson rule for a given set of values of the parameters ($M_{t\infty}$, T_∞/T_w, m^B/m^A, d^B/d^A). The results are shown in Tables I and II. With the help of these data, one can immediately evaluate p_∞/p_w and $\hat{\mathcal{N}}_f$.

The parameter relation and the total particle-flow rate thus evaluated are shown in Figs. 2–5. To be more specific, in Figs. 2–4, p_∞/p_w of Eq. (126), with the term of $O(M^2_{n\infty})$ being neglected, is shown as a function of $M_{n\infty}$ for various values of Γ and for $M_{t\infty} = 0$, 1, 2, and 3 in the case of $m^B/m^A = 2$ and $d^B/d^A = 1$: Fig. 2 is for $T_\infty/T_w = 0.5$, Fig. 3 for $T_\infty/T_w = 1$, and Fig. 4 for $T_\infty/T_w = 2$. In Fig. 5, $\hat{\mathcal{N}}_f$ of Eq. (134), with the term of $O(M_{n\infty})$ being neglected, is shown for $T_\infty/T_w = 0.5$, 1, 1.5, and 2 in the case of $m^B/m^A = 2$ and $d^B/d^A = 1$. Since $\hat{\mathcal{N}}_f/\Gamma$ is independent of Γ, it is shown as a function of $M_{t\infty}$.

As for the macroscopic quantity, $\hat{n}_{H(0)}$, $\hat{T}_{H(0)}$, $\hat{v}_{2H(0)}$, and $\hat{v}_{1H(1)}$ [$= -(T_w/T_\infty)\hat{T}_{H(0)}$] are independent of Γ, m^B/m^A, and d^B/d^A, while $\hat{n}^B_{H(1)}/\Gamma$ is independent of Γ. In Fig. 6, $\hat{n}_{H(0)}$, $\hat{T}_{H(0)}$, $\hat{v}_{2H(0)}$, and $\hat{v}_{1H(1)}$, as well as $\hat{n}^B_{H(1)}/\Gamma$ in the case of $m^B/m^A = 2$ and $d^B/d^A = 1$, are shown for $M_{t\infty} =$ 0, 1, 2, and 3: Fig. 6(a) is for $T_\infty/T_w = 0.5$, and Fig. 6(b) for $T_\infty/T_w =$ 2.

So far, we have considered the case where the molecules of both components are hard spheres. However, the previous numerical data[4–6] for the function $\mathcal{F}_s$ in Eq. (20a) and the function G_s in Eq. (129a) have been obtained on the basis of the

Table I. Numerical values of $\mathcal{H}_0(T_\infty/T_w, M_{t\infty})$ for various d^B/d^A and m^B/m^A

	$d^B/d^A = 0.5$											
	$m^B/m^A = 0.5$				$m^B/m^A = 1$				$m^B/m^A = 2$			
$\frac{T_\infty}{T_w} \setminus M_{t\infty}$	0	1	2	3	0	1	2	3	0	1	2	3
0.5	2.5251	2.5129	2.4787	2.4288	1.8738	1.8547	1.8020	1.7267	1.4710	1.4470	1.3820	1.2914
1	2.6820	2.6632	2.6124	2.5420	2.1395	2.1073	2.0221	1.9080	1.8299	1.7855	1.6706	1.5221
1.5	2.7620	2.7387	2.6771	2.5944	2.2871	2.2452	2.1373	1.9983	2.0486	1.9876	1.8348	1.6464
2	2.8132	2.7865	2.7170	2.6259	2.3867	2.3371	2.2118	2.0549	2.2048	2.1300	1.9467	1.7283
	$d^B/d^A = 1$											
	$m^B/m^A = 0.5$				$m^B/m^A = 1$				$m^B/m^A = 2$			
$\frac{T_\infty}{T_w} \setminus M_{t\infty}$	0	1	2	3	0	1	2	3	0	1	2	3
0.5	1.3855	1.3772	1.3541	1.3204	0.97662	0.96309	0.92610	0.87406	0.71462	0.69803	0.65366	0.59354
1	1.5086	1.4951	1.4589	1.4092	1.2034	1.1783	1.1127	1.0268	1.0293	0.99333	0.90217	0.78893
1.5	1.5741	1.5569	1.5118	1.4521	1.3400	1.3055	1.2182	1.1087	1.2479	1.1940	1.0626	0.90810
2	1.6170	1.5969	1.5452	1.4783	1.4365	1.3943	1.2895	1.1624	1.4171	1.3471	1.1808	0.99286
	$d^B/d^A = 2$											
	$m^B/m^A = 0.5$				$m^B/m^A = 1$				$m^B/m^A = 2$			
$\frac{T_\infty}{T_w} \setminus M_{t\infty}$	0	1	2	3	0	1	2	3	0	1	2	3
0.5	0.59183	0.58799	0.57734	0.56188	0.36124	0.35454	0.33644	0.31145	0.21501	0.20780	0.18897	0.16460
1	0.67050	0.66361	0.64521	0.62013	0.53487	0.51921	0.47911	0.42826	0.45748	0.43374	0.37602	0.30905
1.5	0.71558	0.70627	0.68207	0.65038	0.65985	0.63545	0.57519	0.50281	0.69087	0.64613	0.54239	0.43014
2	0.74650	0.73520	0.70641	0.66979	0.75861	0.72599	0.64763	0.55713	0.91279	0.84464	0.69218	0.53528

Table II. Numerical values of $\mathcal{H}_1(T_\infty/T_w, M_{t\infty})$ for various d^B/d^A and m^B/m^A

	$d^B/d^A = 0.5$											
	$m^B/m^A = 0.5$				$m^B/m^A = 1$				$m^B/m^A = 2$			
$\frac{T_\infty}{T_w} \setminus M_{t\infty}$	0	1	2	3	0	1	2	3	0	1	2	3
0.5	0.65355	0.64973	0.63910	0.62365	0.54973	0.54350	0.52640	0.50208	0.46374	0.45574	0.43405	0.40396
1	0.72841	0.72170	0.70373	0.67915	0.68147	0.66936	0.63751	0.59533	0.64663	0.62923	0.58440	0.52704
1.5	0.77052	0.76157	0.73822	0.70750	0.76273	0.74564	0.70209	0.64695	0.77117	0.74511	0.68027	0.60147
2	0.79909	0.78831	0.76075	0.72549	0.82103	0.79970	0.74654	0.68138	0.86622	0.83231	0.75014	0.65394
	$d^B/d^A = 1$											
	$m^B/m^A = 0.5$				$m^B/m^A = 1$				$m^B/m^A = 2$			
$\frac{T_\infty}{T_w} \setminus M_{t\infty}$	0	1	2	3	0	1	2	3	0	1	2	3
0.5	0.53453	0.53143	0.52282	0.51027	0.41702	0.41170	0.39711	0.37648	0.32228	0.31555	0.29744	0.27268
1	0.60138	0.59579	0.58083	0.56037	0.54617	0.53505	0.50600	0.46787	0.50723	0.49051	0.44803	0.39492
1.5	0.63970	0.63213	0.61240	0.58645	0.63034	0.61404	0.57279	0.52120	0.64727	0.62033	0.55446	0.47670
2	0.66600	0.65679	0.63327	0.60321	0.69286	0.67195	0.62032	0.55795	0.76153	0.72468	0.63719	0.53809
	$d^B/d^A = 2$											
	$m^B/m^A = 0.5$				$m^B/m^A = 1$				$m^B/m^A = 2$			
$\frac{T_\infty}{T_w} \setminus M_{t\infty}$	0	1	2	3	0	1	2	3	0	1	2	3
0.5	0.34902	0.34711	0.34176	0.33392	0.22738	0.22384	0.21416	0.20057	0.13975	0.13580	0.12536	0.11152
1	0.40138	0.39771	0.38787	0.37432	0.34848	0.33954	0.31641	0.28656	0.31389	0.29965	0.26452	0.22272
1.5	0.43264	0.42750	0.41405	0.39629	0.43967	0.42509	0.38873	0.34432	0.49030	0.46199	0.39535	0.32136
2	0.45468	0.44826	0.43184	0.41080	0.51391	0.49379	0.44499	0.38775	0.66368	0.61892	0.51726	0.40995

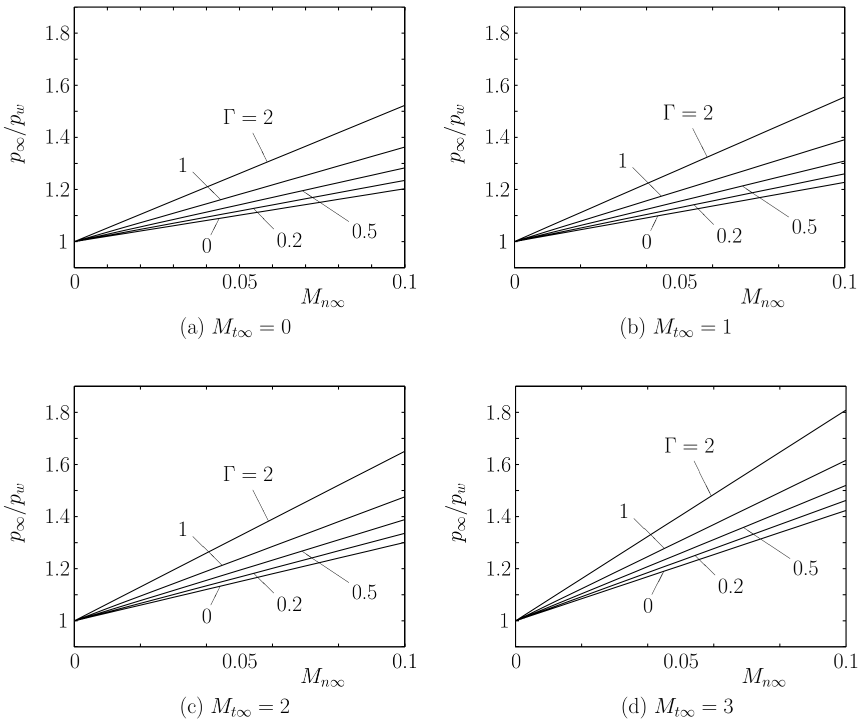

Fig. 2. p_∞/p_w vs $M_{n\infty}$ for $T_\infty/T_w = 0.5$ in the case of $m^B/m^A = 2$ and $d^B/d^A = 1$. (a) $M_{t\infty} = 0$, (b) $M_{t\infty} = 1$, (c) $M_{t\infty} = 2$, (d) $M_{t\infty} = 3$.

GSB model (under the restriction that the molecules of the vapor are mechanically identical with those of the noncondensable gas). Therefore, we summarize the corresponding results for the GSB model in Appendix E. In Fig. 7, we show p_∞/p_w of Eq. (E13), with the term of $O(M_{n\infty}^2)$ being neglected, as a function of $M_{n\infty}$ for various Γ and for $M_{t\infty} = 0$, 1, 2, and 3 in the case of $T_\infty/T_w = 1$, $m^B/m^A = 2$, and $C^{AB}/C^{AA} = C^{BB}/C^{AA} = 1$. In the figure, the result obtained by the direct numerical solution of the original boundary-value problem, Eqs. (E1), (10a), (10b), (12a), and (12b), is also shown. On the other hand, in Fig. 8, $\hat{\mathcal{N}}_f$ of Eq. (E14), with the term of $O(M_{n\infty})$ being neglected, is compared with the result of the direct numerical solution in the same case as in Fig. 7 (except the trivial case of $M_{t\infty} = 0$).

10. CONCLUDING REMARKS

We have considered the half-space problem of condensing vapor flows in the presence of a noncondensable gas. The problem itself is a fundamental problem

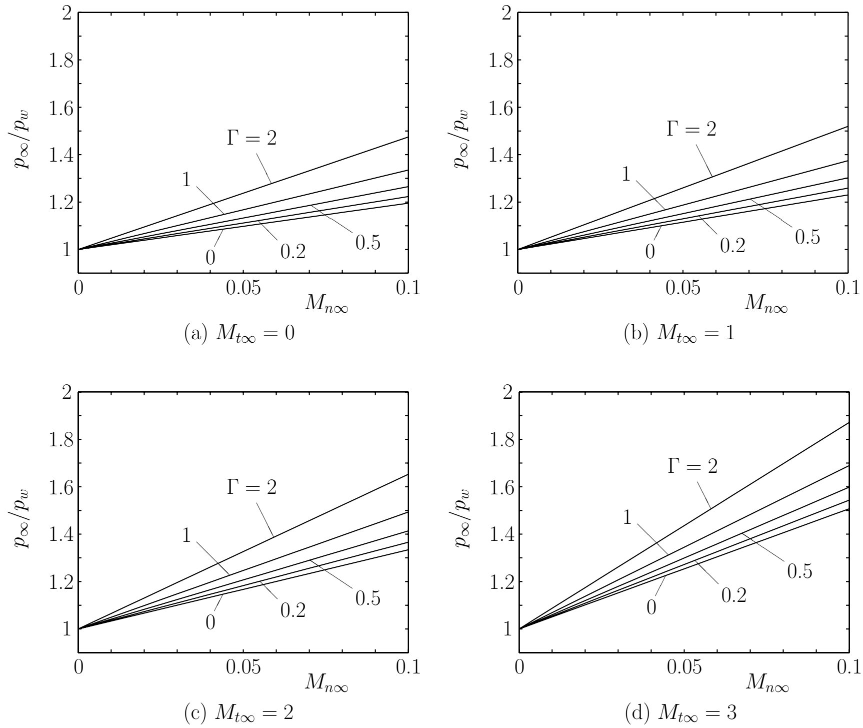

Fig. 3. p_∞/p_w vs $M_{n\infty}$ for $T_\infty/T_w = 1$ in the case of $m^B/m^A = 2$ and $d^B/d^A = 1$. (a) $M_{t\infty} = 0$, (b) $M_{t\infty} = 1$, (c) $M_{t\infty} = 2$, (d) $M_{t\infty} = 3$.

for the Boltzmann equation that deserves rigorous mathematical study. Another important aspect of the problem is a generator for the boundary condition for the compressible Euler equation for the vapor flows in the continuum limit in the presence of a trace of a noncondensable gas, as described in Sec. 1. More specifically, the boundary condition on the condensing surface is essentially given by Eqs. (20) and (129) with an auxiliary condition.[3,37] In refs. 6 and 7, the functions $\mathcal{F}_s$ and $\mathcal{F}_b$ in Eq. (20) and G_s and G_b in Eq. (129) were constructed numerically. However, the construction of functions of three or four independent variables by solving the Boltzmann equation each time is a formidable task, so that the following two simplifications were introduced: (i) Only the case where the mechanical property of the vapor molecules is the same as that of the noncondensable gas was considered; (ii) in place of the Boltzmann equation, the GSB model was employed.

In the present study, restricting ourselves to the case of slow condensation ($M_{n\infty} \ll 1$), we tried to release the restrictions (i) and (ii). That is, we have derived analytical expressions of the functions $\mathcal{F}_s$ and G_s for hard-sphere molecules on the

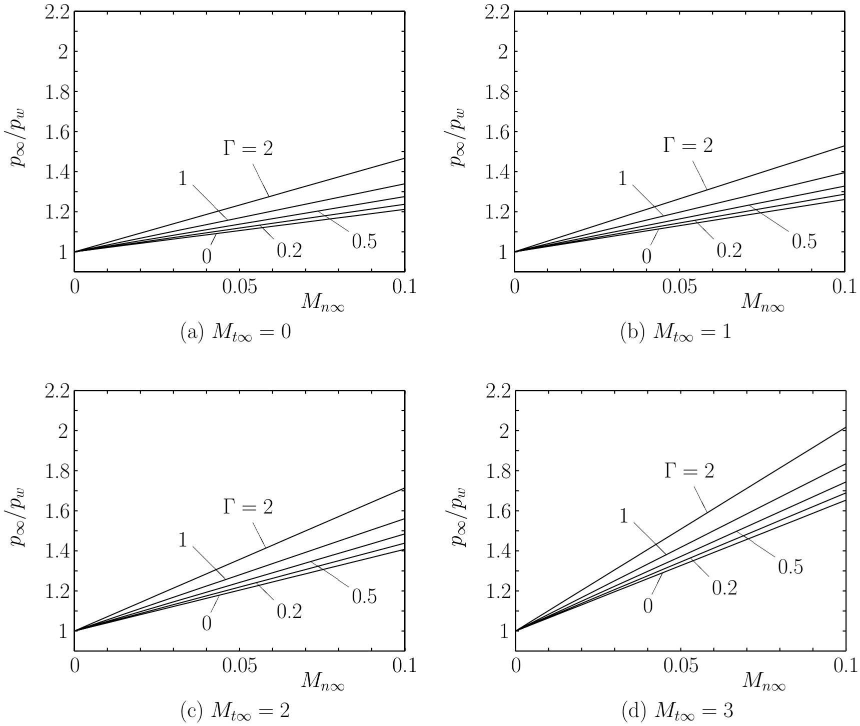

Fig. 4. p_∞/p_w vs $M_{n\infty}$ for $T_\infty/T_w = 2$ in the case of $m^B/m^A = 2$ and $d^B/d^A = 1$. (a) $M_{t\infty} = 0$, (b) $M_{t\infty} = 1$, (c) $M_{t\infty} = 2$, (d) $M_{t\infty} = 3$.

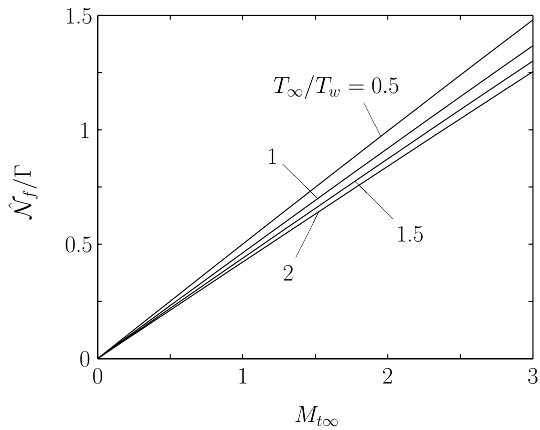

Fig. 5. $\hat{\mathcal{N}}_f/\Gamma$ vs $M_{t\infty}$ for various T_∞/T_w in the case of $m^B/m^A = 2$ and $d^B/d^A = 1$.

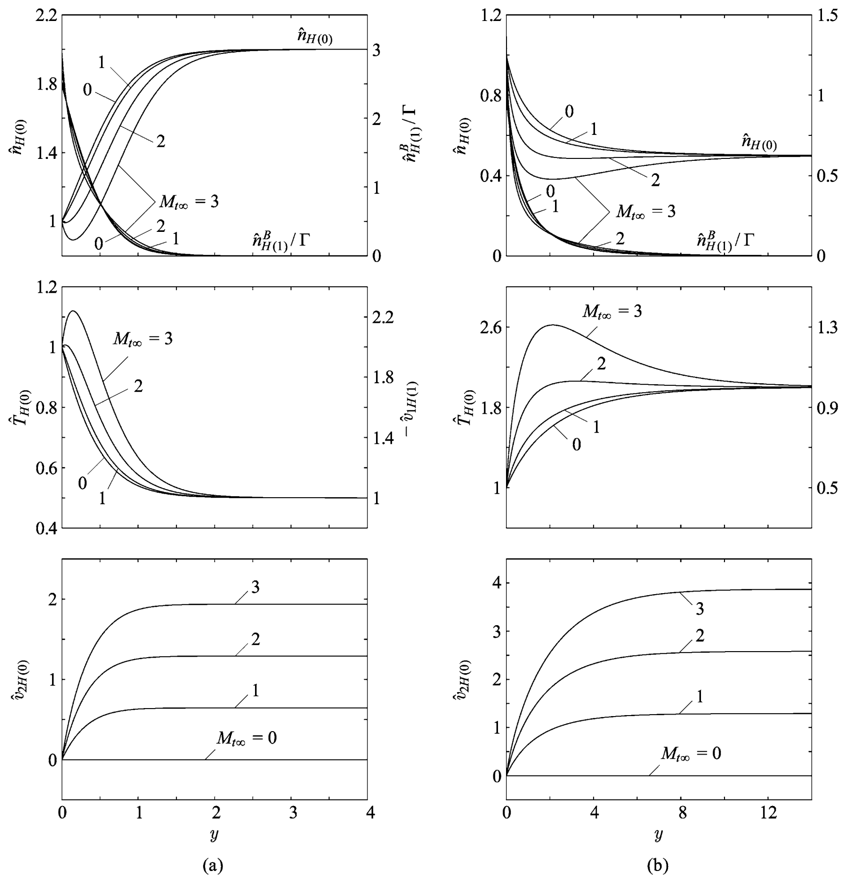

Fig. 6. Profiles of the macroscopic quantities for various $M_{t\infty}$. (a) $T_\infty/T_w = 0.5$, (b) $T_\infty/T_w = 2$. The $\hat{n}_{H(0)}$, $\hat{T}_{H(0)}$, $\hat{v}_{2H(0)}$, and $\hat{v}_{1H(1)}$ $[= -(T_w/T_\infty)T_{H(0)}]$ are independent of Γ, m^B/m^A, and d^B/d^A, while $\hat{n}^B_{H(1)}/\Gamma$ is independent of Γ; $\hat{n}^B_{H(1)}/\Gamma$ for $m^B/m^A = 2$ and $d^B/d^A = 1$ is shown in the figure.

basis of the Boltzmann equation. The result is given by Eqs. (126) and (134). These expressions can be used conveniently in the boundary conditions for the Euler equations on the condensing surface. In addition, we have clarified the behavior of the vapor flows passing through the noncondensable gas and condensing onto the plane condensed phase by obtaining the analytical expressions of the macroscopic quantities.

APPENDIX A: SOME PROPERTIES OF COLLISION INTEGRAL

The collision integral $\hat{J}^{\beta\alpha}(f, g)$ defined in Eq. (9a) has the following properties[(22–24)]:

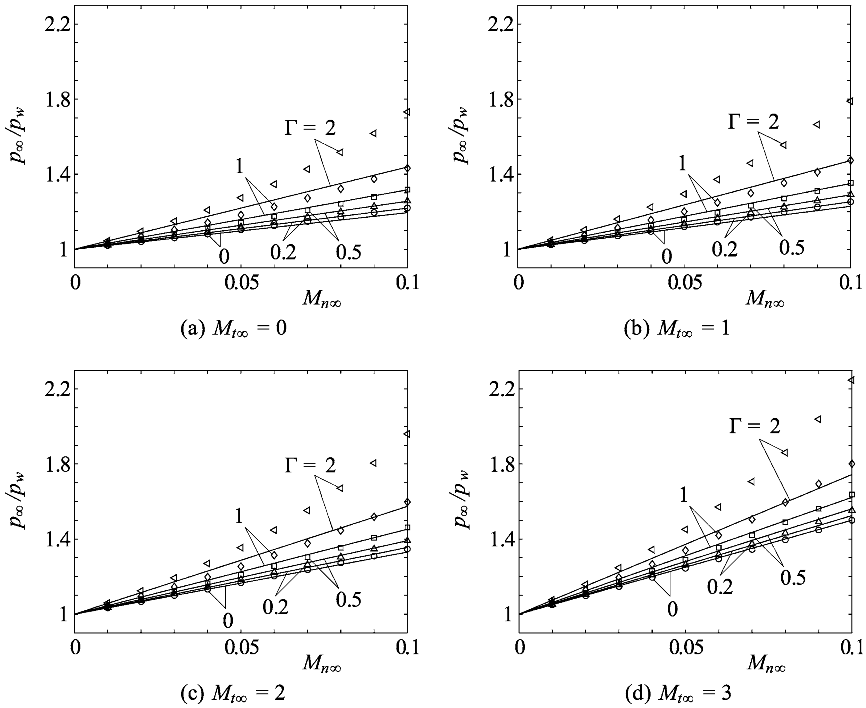

Fig. 7. p_∞/p_w vs $M_{n\infty}$ for the GSB model ($T_\infty/T_w = 1$, $m^B/m^A = 2$, and $C^{AB}/C^{AA} = C^{BB}/C^{AA} = 1$). (a) $M_{t\infty} = 0$, (b) $M_{t\infty} = 1$, (c) $M_{t\infty} = 2$, (d) $M_{t\infty} = 3$. The symbols ○ ($\Gamma = 0$), △ ($\Gamma = 0.2$), □ ($\Gamma = 0.5$), ◇ ($\Gamma = 1$), and ◁ ($\Gamma = 2$) indicate the results of direct numerical analysis of the GSB model.

$$\int \hat{J}^{\beta\alpha}(f, g) d^3\zeta = 0, \tag{A1}$$

$$\int \begin{pmatrix} \zeta_i \\ \zeta_j^2 \end{pmatrix} [\hat{J}^{\alpha\alpha}(f, g) + \hat{J}^{\alpha\alpha}(g, f)] d^3\zeta = 0, \tag{A2}$$

$$\int \begin{pmatrix} \zeta_i \\ \zeta_j^2 \end{pmatrix} [\hat{m}^A \hat{K}^{BA} \hat{J}^{BA}(f, g) + \hat{m}^B \hat{K}^{AB} \hat{J}^{AB}(g, f)] d^3\zeta = 0, \tag{A3}$$

where $d^3\zeta = d\zeta_1 d\zeta_2 d\zeta_3$, and the domain of integration is the whole space of ζ_i ($\hat{m}^A = 1$).

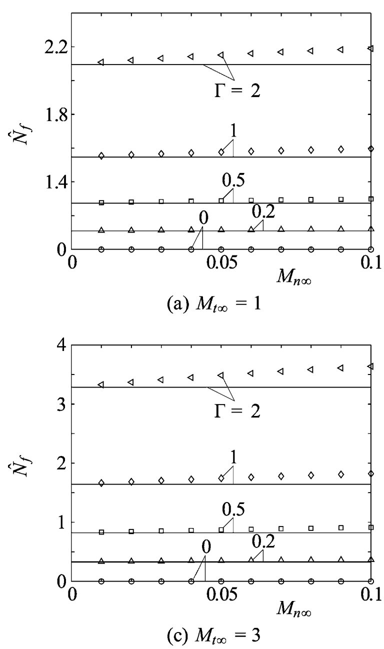

Fig. 8. $\hat{\mathcal{N}}_f$ vs $M_{n\infty}$ for the GSB model ($T_\infty/T_w = 1$, $m^B/m^A = 2$, and $C^{AB}/C^{AA} = C^{BB}/C^{AA} = 1$). (a) $M_{t\infty} = 1$, (b) $M_{t\infty} = 2$, (c) $M_{t\infty} = 3$. See the caption of Fig. 7.

APPENDIX B: EXPLICIT FORM OF $h_{H(m)}$

$$\hat{n}^\alpha_{H(0)} = \int \hat{F}^\alpha_{H(0)} d^3\zeta, \qquad \hat{\rho}^\alpha_{H(0)} = \hat{m}^\alpha \hat{n}^\alpha_{H(0)}, \tag{B1a}$$

$$\hat{v}^\alpha_{iH(0)} = \frac{1}{\hat{n}^\alpha_{H(0)}} \int \zeta_i \hat{F}^\alpha_{H(0)} d^3\zeta, \tag{B1b}$$

$$\hat{p}^\alpha_{H(0)} = \hat{n}^\alpha_{H(0)} \hat{T}^\alpha_{H(0)} = \frac{2}{3}\hat{m}^\alpha \int \left(\zeta_j - \hat{v}^\alpha_{jH(0)}\right)^2 \hat{F}^\alpha_{H(0)} d^3\zeta, \tag{B1c}$$

$$\hat{n}_{H(0)} = \sum_{\alpha=A,B} \hat{n}^\alpha_{H(0)}, \qquad \hat{\rho}_{H(0)} = \sum_{\alpha=A,B} \hat{\rho}^\alpha_{H(0)}, \tag{B1d}$$

$$\hat{\rho}_{H(0)}\hat{v}_{iH(0)} = \sum_{\alpha=A,B} \hat{\rho}^\alpha_{H(0)} \hat{v}^\alpha_{iH(0)}, \tag{B1e}$$

$$\hat{p}_{H(0)} = \hat{n}_{H(0)}\hat{T}_{H(0)} = \sum_{\alpha=A,B}\left[\hat{p}^\alpha_{H(0)} + \frac{2}{3}\hat{\rho}^\alpha_{H(0)}(\hat{v}^\alpha_{jH(0)} - \hat{v}_{jH(0)})^2\right], \tag{B1f}$$

$$\hat{n}^{\alpha}_{H(1)} = \int \hat{F}^{\alpha}_{H(1)} d^3\zeta, \qquad \hat{\rho}^{\alpha}_{H(1)} = \hat{m}^{\alpha}\hat{n}^{\alpha}_{H(1)}, \tag{B2a}$$

$$\hat{v}^{\alpha}_{iH(1)} = \frac{1}{\hat{n}^{\alpha}_{H(0)}} \int (\zeta_i - \hat{v}^{\alpha}_{iH(0)}) \hat{F}^{\alpha}_{H(1)} d^3\zeta, \tag{B2b}$$

$$\hat{T}^{\alpha}_{H(1)} = \frac{2\hat{m}^{\alpha}}{3\hat{n}^{\alpha}_{H(0)}} \int (\zeta_j - \hat{v}^{\alpha}_{jH(0)})^2 \hat{F}^{\alpha}_{H(1)} d^3\zeta - \left(\frac{\hat{n}^{\alpha}_{H(1)}}{\hat{n}^{\alpha}_{H(0)}}\right) \hat{T}^{\alpha}_{H(0)}, \tag{B2c}$$

$$\hat{p}^{\alpha}_{H(1)} = \hat{n}^{\alpha}_{H(0)}\hat{T}^{\alpha}_{H(1)} + \hat{n}^{\alpha}_{H(1)}\hat{T}^{\alpha}_{H(0)}, \tag{B2d}$$

$$\hat{n}_{H(1)} = \sum_{\alpha=A,B} \hat{n}^{\alpha}_{H(1)}, \qquad \hat{\rho}_{H(1)} = \sum_{\alpha=A,B} \hat{\rho}^{\alpha}_{H(1)}, \tag{B2e}$$

$$\hat{v}_{iH(1)} = \frac{1}{\hat{\rho}_{H(0)}} \sum_{\alpha=A,B} (\hat{\rho}^{\alpha}_{H(0)}\hat{v}^{\alpha}_{iH(1)} + \hat{\rho}^{\alpha}_{H(1)}\hat{v}^{\alpha}_{iH(0)}) - \frac{\hat{\rho}_{H(1)}}{\hat{\rho}_{H(0)}} \hat{v}_{iH(0)}, \tag{B2f}$$

$$\hat{p}_{H(1)} = \sum_{\alpha=A,B} \left[\hat{p}^{\alpha}_{H(1)} + \frac{2}{3}\hat{\rho}^{\alpha}_{H(1)} (\hat{v}^{\alpha}_{jH(0)} - \hat{v}_{jH(0)})^2 \right.$$
$$\left. + \frac{4}{3}\hat{\rho}^{\alpha}_{H(0)} (\hat{v}^{\alpha}_{jH(0)} - \hat{v}_{jH(0)})(\hat{v}^{\alpha}_{jH(1)} - \hat{v}_{jH(1)}) \right], \tag{B2g}$$

$$\hat{p}_{H(1)} = \hat{n}_{H(1)}\hat{T}_{H(0)} + \hat{n}_{H(0)}\hat{T}_{H(1)}. \tag{B2h}$$

$$\hat{n}^{\alpha}_{H(2)} = \int \hat{F}^{\alpha}_{H(2)} d^3\zeta, \qquad \hat{\rho}^{\alpha}_{H(2)} = \hat{m}^{\alpha}\hat{n}^{\alpha}_{H(2)}, \tag{B3a}$$

$$\hat{v}^{\alpha}_{iH(2)} = \frac{1}{\hat{n}^{\alpha}_{H(0)}} \int (\zeta_i - \hat{v}^{\alpha}_{iH(0)}) \hat{F}^{\alpha}_{H(2)} d^3\zeta - \left(\frac{\hat{n}^{\alpha}_{H(1)}}{\hat{n}^{\alpha}_{H(0)}}\right) \hat{v}^{\alpha}_{iH(1)}, \tag{B3b}$$

$$\hat{T}^{\alpha}_{H(2)} = \frac{2\hat{m}^{\alpha}}{3\hat{n}^{\alpha}_{H(0)}} \int (\zeta_j - \hat{v}^{\alpha}_{jH(0)})^2 \hat{F}^{\alpha}_{H(2)} d^3\zeta$$
$$- \frac{2}{3}\hat{m}^{\alpha}(\hat{v}^{\alpha}_{jH(1)})^2 - \left(\frac{\hat{n}^{\alpha}_{H(1)}}{\hat{n}^{\alpha}_{H(0)}}\right) \hat{T}^{\alpha}_{H(1)} - \left(\frac{\hat{n}^{\alpha}_{H(2)}}{\hat{n}^{\alpha}_{H(0)}}\right) \hat{T}^{\alpha}_{H(0)}, \tag{B3c}$$

$$\hat{p}^{\alpha}_{H(2)} = \hat{n}^{\alpha}_{H(0)}\hat{T}^{\alpha}_{H(2)} + \hat{n}^{\alpha}_{H(1)}\hat{T}^{\alpha}_{H(1)} + \hat{n}^{\alpha}_{H(2)}\hat{T}^{\alpha}_{H(0)}, \tag{B3d}$$

$$\hat{n}_{H(2)} = \sum_{\alpha=A,B} \hat{n}^{\alpha}_{H(2)}, \qquad \hat{\rho}_{H(2)} = \sum_{\alpha=A,B} \hat{\rho}^{\alpha}_{H(2)}, \tag{B3e}$$

$$\hat{v}_{iH(2)} = \frac{1}{\hat{\rho}_{H(0)}} \sum_{\alpha=A,B} (\hat{\rho}^{\alpha}_{H(0)}\hat{v}^{\alpha}_{iH(2)} + \hat{\rho}^{\alpha}_{H(1)}\hat{v}^{\alpha}_{iH(1)} + \hat{\rho}^{\alpha}_{H(2)}\hat{v}^{\alpha}_{iH(0)}) - \frac{\hat{\rho}_{H(1)}}{\hat{\rho}_{H(0)}}\hat{v}_{iH(1)} - \frac{\hat{\rho}_{H(2)}}{\hat{\rho}_{H(0)}}\hat{v}_{iH(0)}, \tag{B3f}$$

$$\hat{p}_{H(2)} = \sum_{\alpha=A,B} \Big\{ \hat{p}^{\alpha}_{H(2)} + \frac{2}{3}\hat{\rho}^{\alpha}_{H(0)}[(\hat{v}^{\alpha}_{jH(1)} - \hat{v}_{jH(1)})^2 + 2(\hat{v}^{\alpha}_{jH(0)} - \hat{v}_{jH(0)})(\hat{v}^{\alpha}_{jH(2)} - \hat{v}_{jH(2)})] + \frac{4}{3}\hat{\rho}^{\alpha}_{H(1)}(\hat{v}^{\alpha}_{jH(0)} - \hat{v}_{jH(0)})(\hat{v}^{\alpha}_{jH(1)} - \hat{v}_{jH(1)}) + \frac{2}{3}\hat{\rho}^{\alpha}_{H(2)}(\hat{v}^{\alpha}_{jH(0)} - \hat{v}_{jH(0)})^2 \Big\}, \tag{B3g}$$

$$\hat{p}_{H(2)} = \hat{n}_{H(0)}\hat{T}_{H(2)} + \hat{n}_{H(1)}\hat{T}_{H(1)} + \hat{n}_{H(2)}\hat{T}_{H(0)}. \tag{B3h}$$

APPENDIX C: FUNCTIONS $A(\zeta)$ AND $B(\zeta)$

The functions $A(\zeta)$ and $B(\zeta)$ in Eq. (49) are the solution of the following integral equations:

$$\widetilde{L}^{AA}(\zeta_i A(\zeta), \zeta_i A(\zeta)) = -\zeta_i(\zeta^2 - 5/2), \tag{C1a}$$

$$\text{subsidiary condition:} \int \zeta^4 A(\zeta) E^A(\zeta) d\zeta = 0, \tag{C1b}$$

$$\widetilde{L}^{AA}\left(\left(\zeta_i\zeta_j - \frac{1}{3}\zeta^2\delta_{ij}\right)B(\zeta),\ \left(\zeta_i\zeta_j - \frac{1}{3}\zeta^2\delta_{ij}\right)B(\zeta)\right) = -2\left(\zeta_i\zeta_j - \frac{1}{3}\zeta^2\delta_{ij}\right), \tag{C2}$$

where

$$\zeta = (\zeta_j^2)^{1/2}, \quad E^{\alpha}(\zeta) = \left(\frac{\hat{m}^{\alpha}}{\pi}\right)^{3/2} \exp(-\hat{m}^{\alpha}\zeta^2), \tag{C3}$$

δ_{ij} is the Kronecker delta, and $\widetilde{L}^{\beta\alpha}(f, g)$ is the linearized collision operator defined by

$$\begin{aligned}\widetilde{L}^{\beta\alpha}(f, g) &= [\hat{J}^{\beta\alpha}(fE^{\beta}, E^{\alpha}) + \hat{J}^{\beta\alpha}(E^{\beta}, gE^{\alpha})]/E^{\alpha}(\zeta) \\ &= \frac{1}{4\sqrt{2\pi}} \int E^{\beta}(\zeta_*)[f(\zeta_{*i}^{\beta\alpha}) + g(\zeta_i^{\beta\alpha}) \\ &\quad - f(\zeta_{*i}) - g(\zeta_i)]|e_j \hat{V}_j| d\Omega(e_i) d^3\zeta_*, \qquad \text{(C4)}\end{aligned}$$

with $\zeta_* = (\zeta_{*j}^2)^{1/2}$. The $\widetilde{L}^{AA}(f, f)$ is identical to the linearized collision operator for a single-component gas. The functions $A(\zeta)$ and $B(\zeta)$, which were obtained numerically in refs. 38 and 39, are given in Table 3.1 in ref. 2.

The γ_1 and γ_2 appearing in Eqs. (52c) and (52d) [or Eqs. (81c) and (81d)] are defined by

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \frac{8\pi}{15} \int_0^{\infty} \begin{bmatrix} B(\zeta) \\ 2A(\zeta) \end{bmatrix} \zeta^6 E^A(\zeta)\, d\zeta, \qquad \text{(C5)}$$

in terms of $B(\zeta)$ and $A(\zeta)$ and have numerical values[2,39]

$$\gamma_1 = 1.270042427, \qquad \gamma_2 = 1.922284066. \qquad \text{(C6)}$$

The γ_1 and γ_2 are related to the viscosity μ and the thermal conductivity λ of the vapor at temperature T as

$$\mu = \frac{\gamma_1 m^A (2kT/m^A)^{1/2}}{4\sqrt{2\pi}(d^A)^2}, \qquad \lambda = \frac{5\gamma_2 k (2kT/m^A)^{1/2}}{8\sqrt{2\pi}(d^A)^2}. \qquad \text{(C7)}$$

Table III. Numerical values of $\hat{\Delta}^*_{BA} = \hat{\Delta}_{BA}(X^A = 1)$ and $\hat{D}^*_{TB} = \hat{D}_{TB}(X^A = 1)$ for various d^B/d^A and m^B/m^A

	$d^B/d^A = 0.5$		$d^B/d^A = 1$		$d^B/d^A = 2$	
m^B/m^A	$\hat{\Delta}^*_{BA}$	$\hat{D}^*_{TB}$	$\hat{\Delta}^*_{BA}$	$\hat{D}^*_{TB}$	$\hat{\Delta}^*_{BA}$	$\hat{D}^*_{TB}$
0.1	−0.34449	−1.5877	−0.19377	−0.90478	−0.086121	−0.41699
0.2	−0.49896	−1.0531	−0.28066	−0.60782	−0.12474	−0.28974
0.5	−0.85156	−0.48449	−0.47900	−0.26582	−0.21289	−0.10963
1	−1.3586	−0.10757	−0.764215339	0	−0.33965	0.076834
2	−2.3241	0.16818	−1.3073	0.21582	−0.58102	0.24985
3	−3.2761	0.27340	−1.8428	0.30276	−0.81903	0.32374
4	−4.2241	0.32771	−2.3760	0.34866	−1.0560	0.36363
5	−5.1702	0.36065	−2.9082	0.37687	−1.2925	0.38845
8	−8.0039	0.41042	−4.5022	0.42000	−2.0010	0.42685
10	−9.8914	0.42706	−5.5639	0.43457	−2.4728	0.43994

APPENDIX D: FUNCTIONS $A^B(\zeta; X^A)$, $B^B(\zeta; X^A)$, AND $D^{(A)B}(\zeta; X^A)$

The functions $A^\alpha(\zeta; X^A)$, $B^\alpha(\zeta; X^A)$, and $D^{(\gamma)\alpha}(\zeta; X^A)$ ($\alpha = A, B$, $\gamma = A, B$) are, respectively, the solutions of the following integral equations[27]:

$$\sum_{\beta=A,B} \hat{K}^{\beta\alpha} X^\beta \widetilde{L}^{\beta\alpha} \left(\zeta_i A^\beta(\zeta), \zeta_i A^\alpha(\zeta)\right) = -\zeta_i \left(\hat{m}^\alpha \zeta^2 - \frac{5}{2}\right), \tag{D1a}$$

$$\text{subsidiary condition:} \sum_{\beta=A,B} \hat{m}^\beta X^\beta \int \zeta^4 A^\beta(\zeta) E^\beta(\zeta) d\zeta = 0, \tag{D1b}$$

$$\sum_{\beta=A,B} \hat{K}^{\beta\alpha} X^\beta \widetilde{L}^{\beta\alpha} \left(\left(\zeta_i\zeta_j - \frac{1}{3}\zeta^2\delta_{ij}\right) B^\beta(\zeta), \left(\zeta_i\zeta_j - \frac{1}{3}\zeta^2\delta_{ij}\right) B^\alpha(\zeta)\right)$$
$$= -2\hat{m}^\alpha \left(\zeta_i\zeta_j - \frac{1}{3}\zeta^2\delta_{ij}\right), \tag{D2}$$

$$\sum_{\beta=A,B} \hat{K}^{\beta\alpha} X^\alpha X^\beta \widetilde{L}^{\beta\alpha} \left(\zeta_i D^{(\gamma)\beta}(\zeta), \zeta_i D^{(\gamma)\alpha}(\zeta)\right) = -\zeta_i \left(\delta_{\alpha\gamma} - \frac{\hat{m}^\alpha X^\alpha}{\sum \hat{m}^\beta X^\beta}\right), \tag{D3a}$$

$$\text{subsidiary condition:} \sum_{\beta=A,B} \hat{m}^\beta X^\beta \int \zeta^4 D^{(\gamma)\beta}(\zeta) E^\beta(\zeta) d\zeta = 0. \tag{D3b}$$

Here, X^A is a parameter ($0 \le X^A \le 1$) and $X^B = 1 - X^A$. Physically, X^A indicates the concentration of the A-component, and thus X^B that of the B-component. In Eqs. (D1a)–(D3b), the dependence of X^A in A^α, B^α, and $D^{(\gamma)\alpha}$ is omitted for simplicity. In Eq. (54), only A^B, B^B, and $D^{(A)B}$ at $X^A = 1$ appear.

Let us introduce the following $\hat{\Delta}_{\alpha\beta}$ and $\hat{D}_{T\alpha}$[27]:

$$\begin{bmatrix} \hat{\Delta}_{\alpha\beta}(X^A) \\ \hat{D}_{T\alpha}(X^A) \end{bmatrix} = \frac{4\pi}{3} \int_0^\infty \zeta^4 \begin{bmatrix} D^{(\beta)\alpha}(\zeta; X^A) \\ A^\alpha(\zeta; X^A) \end{bmatrix} E^\alpha(\zeta) d\zeta. \tag{D4}$$

Then, $\hat{\Delta}^*_{BA}$ and $\hat{D}^*_{TB}$ occurring in Eq. (56) are defined by

$$\hat{\Delta}^*_{BA} = \hat{\Delta}_{BA}(X^A = 1), \qquad \hat{D}^*_{TB} = \hat{D}_{TB}(X^A = 1). \tag{D5}$$

In ref. 28, the database for the various transport coefficients for a binary mixture of hard-sphere gases has been constructed. In this process, the functions $A^\alpha(\zeta; X^A)$ and $D^{(\gamma)\alpha}(\zeta; X^A)$ have been obtained though their data are not shown in the paper. If the data are exploited, the integration in Eq. (D4) is immediate. Some of the numerical results are shown in Table III.

APPENDIX E: RESULTS FOR THE GSB MODEL

To start with, we show the GSB model in the dimensionless form. The dimensionless variables are the same as those used for the hard-sphere molecules [cf. Eqs. (6) and (7)]. The GSB model in the present steady and spatially one dimensional case is written as follows:

$$\zeta_1 \frac{\partial \hat{F}^\alpha}{\partial x_1} = \sum_{\beta=A,B} \hat{C}^{\beta\alpha} \hat{n}^\beta (\hat{F}^{\beta\alpha} - \hat{F}^\alpha), \tag{E1}$$

where

$$\begin{aligned}\hat{F}^{\beta\alpha} &= \hat{n}^\alpha \left(\frac{\hat{m}^\alpha}{\pi \hat{T}}\right)^{3/2} \exp\left(-\frac{\hat{m}^\alpha(\zeta_j - \hat{v}_j)^2}{\hat{T}}\right) \\ &\quad \times \left\{1 + 2\frac{\hat{m}^\alpha}{\hat{T}}(\hat{v}_j^{\beta\alpha} - \hat{v}_j)(\zeta_j - \hat{v}_j) \right. \\ &\quad \left. + \left[\frac{\hat{T}^{\beta\alpha} - \hat{T}}{\hat{T}} + \frac{2}{3}\frac{\hat{m}^\alpha}{\hat{T}}(\hat{v}_j^{\beta\alpha} - \hat{v}_j)^2\right]\left[\frac{\hat{m}^\alpha(\zeta_j - \hat{v}_j)^2}{\hat{T}} - \frac{3}{2}\right]\right\},\end{aligned} \tag{E2a}$$

$$\hat{v}_i^{\beta\alpha} = \frac{\hat{m}^\alpha \hat{v}_i^\alpha + \hat{m}^\beta \hat{v}_i^\beta}{\hat{m}^\alpha + \hat{m}^\beta}, \tag{E2b}$$

$$\hat{T}^{\beta\alpha} = \frac{\hat{m}^\alpha \hat{m}^\beta}{(\hat{m}^\alpha + \hat{m}^\beta)^2}\left[\left(\frac{\hat{m}^\alpha}{\hat{m}^\beta} + \frac{\hat{m}^\beta}{\hat{m}^\alpha}\right)\hat{T}^\alpha + 2\hat{T}^\beta + \frac{2}{3}\hat{m}^\beta(\hat{v}_j^\alpha - \hat{v}_j^\beta)^2\right], \tag{E2c}$$

$$\hat{C}^{\beta\alpha} = C^{\beta\alpha}/C^{AA}, \tag{E2d}$$

and $C^{\beta\alpha}$ is a constant ($C^{AB} = C^{BA}$). The $\hat{n}^\alpha$, $\hat{v}_i^\alpha$, $\hat{T}^\alpha$, $\hat{v}_i$, and $\hat{T}$ in Eqs. (E1)–(E2c) are given by Eqs. (13a)–(13f). The collision frequency $\nu^{\beta\alpha}$ of the α-molecules for their collisions with the β-molecules is given by $\nu^{\beta\alpha} = C^{\beta\alpha} n^\beta$. Therefore, the mean free paths l_w and l_∞ of the vapor molecules [cf. Eqs. (5) and (16)] are given respectively by

$$l_w = \frac{2}{\sqrt{\pi}}\left(\frac{2kT_w}{m^A}\right)^{1/2}\frac{1}{C^{AA}n_w}, \qquad l_\infty = \frac{2}{\sqrt{\pi}}\left(\frac{2kT_\infty}{m^A}\right)^{1/2}\frac{1}{C^{AA}n_\infty}. \tag{E3}$$

In this Appendix, the Boltzmann equation (8) is replaced by Eq. (E1). Since $n_w l_w = (T_\infty/T_w)^{-1/2} n_\infty l_\infty$ holds for this model, Γ of Eq. (15) reduces to

$$\Gamma = \hat{T}_\infty^{-1/2} \int_0^\infty \hat{n}^B dx_1, \tag{E4}$$

which is different from Eq. (17) for hard-sphere molecules by the factor $\hat{T}_\infty^{-1/2}$ in the right-hand side. The dimensionless parameters contained in the boundary-value problem, Eqs. (E1), (10a), (10b), (12a), and (12b), are $\hat{T}_\infty$, $\hat{p}_\infty$ (or $\hat{n}_\infty$),

$\hat{v}_{1\infty}$, $\hat{v}_{2\infty}$, $\hat{m}^B$, $\hat{C}^{AB}$, $\hat{C}^{BB}$, and Γ. That is, $\hat{d}^B$ for hard-sphere molecules has been replaced by $\hat{C}^{AB}$ and $\hat{C}^{BB}$.

We have carried out the same asymptotic analysis as in the case of hard-sphere molecules. We give the results listing the difference from the hard-sphere case.

The fluid-dynamic equations and the boundary conditions are the same as Eqs. (81a)–(83), except that

$$\gamma_1 = \hat{T}_{H(0)}^{1/2}, \qquad \gamma_2 = \hat{T}_{H(0)}^{1/2}, \tag{E5a}$$

$$\hat{\Delta}_{BA}^* = -(1+\hat{m}^B)\hat{T}_{H(0)}^{1/2}/2\hat{C}^{AB}, \qquad \hat{D}_{TB}^* = 0, \tag{E5b}$$

$$C_4^* = -2.13204, \qquad C_1 = 0.55844. \tag{E5c}$$

As for the solution of the fluid-dynamic system, Eqs. (87), (89), and (92) hold, first of all.

- **Case I** ($\hat{v}_{2\infty} = 0$ and $\hat{T}_\infty = 1$)

In this case, Eqs. (94)–(98) hold.

- **Case II** ($\hat{v}_{2\infty} = 0$ and $\hat{T}_\infty \neq 1$)

Equation (99) holds. Equations (102), (107), and (109) are replaced respectively by

$$y = -\frac{\hat{T}_\infty}{2}\left(\hat{T}_{H(0)} - 1 + \hat{T}_\infty \ln\left|\frac{\hat{T}_{H(0)} - \hat{T}_\infty}{1-\hat{T}_\infty}\right|\right), \tag{E6}$$

$$\hat{n}_{H(1)}^B = \frac{2\hat{C}^{AB}\hat{m}^B}{1+\hat{m}^B}\frac{\Gamma}{\hat{T}_\infty^{1/2}}\frac{1}{\hat{T}_{H(0)}}\left(\frac{|\hat{T}_{H(0)} - \hat{T}_\infty|}{|1-\hat{T}_\infty|}\right)^{\frac{\hat{C}^{AB}\hat{m}^B}{1+\hat{m}^B}}, \tag{E7}$$

$$\hat{p}_{\infty(1)} = \frac{1}{\hat{T}_\infty}\left[\frac{2\hat{C}^{AB}\hat{m}^B}{1+\hat{m}^B}\hat{T}_\infty^{1/2}\Gamma + 2C_1(\hat{T}_\infty - 1) - C_4^*\right]. \tag{E8}$$

- **Case III** ($\hat{v}_{2\infty} \neq 0$)

Equations (116), (117), (122), and (124) are, respectively, replaced by

$$\hat{T}_{H(0)} = \hat{T}_\infty - \frac{2}{5}\hat{v}_{2\infty}^2 \left(1 - \frac{\hat{v}_{2H(0)}}{\hat{v}_{2\infty}}\right)^2 + \left(1 - \hat{T}_\infty + \frac{2}{5}\hat{v}_{2\infty}^2\right)\left(1 - \frac{\hat{v}_{2H(0)}}{\hat{v}_{2\infty}}\right), \tag{E9}$$

$$y = -\frac{\hat{T}_\infty}{2}\left\{\hat{T}_\infty \ln\left(1 - \frac{\hat{v}_{2H(0)}}{\hat{v}_{2\infty}}\right) + \frac{\hat{v}_{2\infty}^2}{5}\left[1 - \left(1 - \frac{\hat{v}_{2H(0)}}{\hat{v}_{2\infty}}\right)^2\right] - \left(1 - \hat{T}_\infty + \frac{2}{5}\hat{v}_{2\infty}^2\right)\frac{\hat{v}_{2H(0)}}{\hat{v}_{2\infty}}\right\}, \tag{E10}$$

$$\hat{n}^B_{H(1)} = \frac{2\hat{C}^{AB}\hat{m}^B}{1 + \hat{m}^B}\frac{\Gamma}{\hat{T}_\infty^{1/2}}\frac{1}{\hat{T}_{H(0)}}\left(\frac{\hat{v}_{2\infty} - \hat{v}_{2H(0)}}{\hat{v}_{2\infty}}\right)^{\frac{\hat{C}^{AB}\hat{m}^B}{1+\hat{m}^B}}, \tag{E11}$$

$$\hat{p}_{\infty(1)} = \frac{1}{\hat{T}_\infty}\left[\frac{2\hat{C}^{AB}\hat{m}^B}{1 + \hat{m}^B}\hat{T}_\infty^{1/2}\Gamma + 2C_1\left(\hat{T}_\infty - 1 + \frac{2}{5}\hat{v}_{2\infty}^2\right) - C_4^*\right]. \tag{E12}$$

The parameter relation (126) and the dimensionless total particle-flow rate $\hat{\mathcal{N}}_f$ (134) are replaced respectively by

$$\frac{p_\infty}{p_w} = 1 + \left(\frac{5}{6}\right)^{1/2}\left(\frac{T_\infty}{T_w}\right)^{-1/2}\left\{\frac{2m^B/m^A}{1 + m^B/m^A}\frac{C^{AB}}{C^{AA}}\left(\frac{T_\infty}{T_w}\right)^{1/2}\Gamma + 2C_1\left[\frac{T_\infty}{T_w}\left(\frac{M_{t\infty}^2}{3} + 1\right) - 1\right] - C_4^*\right\}M_{n\infty} + O(M_{n\infty}^2), \tag{E13}$$

$$\hat{\mathcal{N}}_f = \left(\frac{5}{6}\right)^{1/2}\frac{1 + m^B/m^A}{1 + (m^B/m^A)(1 + C^{AB}/C^{AA})}\Gamma M_{t\infty} + O(M_{n\infty}). \tag{E14}$$

It should be noted here that $\hat{\mathcal{N}}_f$, which is defined by Eq. (128), becomes

$$\hat{\mathcal{N}}_f = \hat{T}_\infty^{-1}\int_0^\infty \hat{n}^B\hat{v}_2^B\,dx_1, \tag{E15}$$

in place of Eq. (130) for the same reason as the difference between Eqs. (17) and (E4).

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